Permutation & Combination

1. If a and b are the greatest values of ${}^{2n}C_r$ and ${}^{2n-1}C_r$ respectively. Then

(a) a = 2b (b) b = 2a

(c) a = b (d) none of these

2. If n is even and ${}^nC_0 < {}^nC_1 < {}^nC_2 < .... < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > .... > {}^nC_n$, then r =

(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$

(c) $\frac{n-2}{2}$ (d) $\frac{n+2}{2}$

3. $\sum_{k=m}^{n} {}^kC_r$ equals

(a) ${}^{n+1}C_{r+1}$ (b) ${}^{n-1}C_{r+1} - {}^mC_r$

(c) ${}^{n+1}C_{r+1} - {}^nC_{r+1}$ (d) ${}^{n+1}C_{r+1} + {}^mC_{r+1}$

4. The number of signals that can be generated by using 6 differently coloured flags, when any number of them may be hoisted at a time is

(a) 1956 (b) 1957

(c) 1958 (d) 1959

5. The sum of all five digit numbers that can be formed using the digits 1, 2, 3, 4, 5, when reptition of digits is not allowed, is

(a) 366000 (b) 660000

(c) 360000 (d) 3999960

6. Three dice are rolled. The number of possible outcomes in which at least one die shows 5 is

(a) 215 (b) 36

(c) 125 (d) 91

7. The number of times the digit 3 will be writen when listing the integers from 1 to 1000 is

(a) 269 (b) 300

(c) 271 (d) 302

8. The number of ways in which a mixed double game can be arranged from amongst 9 married couples if no husband and wife play in the same game is

(a) 756 (b) 1512

(c) 3024 (d) none of these

9. The number of different seven digit numbers that can be written using only the three digit 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is

(a) ${}^7P_2\ 2^5$ (b) ${}^7C_2\ 2^5$

(c) ${}^7C_2\ 5^2$ (d) none of these

10. In a certain test there are n questions. In this test 2^{n-k} students gave wrong answers to at least k questions, where k = 1, 2, 3, ..., n. If the total number of wrong answers given is 2047, then n is equal to

(a) 10 (b) 11

(c) 12 (d) 13

11. The number of integers which lie between 1 and 106 and which have the sum of digits is equal to 12 is

(a) 8550 (b) 5382

(c) 6062 (d) 8055

12. The number of integral solutions of x + y + z = 0, with $x \geq -5, y \geq -5, z \geq -5$ is

(a) 135 (b) 136

(c) 455 (d) 105

13. The number of ways in which a score of 11 can e made from a throw by three persons, each throwing a single die once, is

(a) 45 (b) 18

(c) 27 (d) none of these

14. If n = ${}^mC_2$, the value of ${}^nC_2$ is given by

(a) ${}^{m+1}C_4$ (b) ${}^{m-1}C_4$

(c) ${}^{m+2}C_4$ (d) none of these

15. The number of positive integers satisfying the inequality ${}^{m+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 100$ is

(a) 9 (b) 8

(c) 5 (d) none of these

16. Out of 10 consonants four vowels, the number of words that can be formed using six consonants and three vowels is

(a) ${}^{10}P_6 \times {}^6P_3$ (b) ${}^{10}C_6 \times {}^6C_3$

(c) ${}^{10}C_6 \times {}^4C_3 \times 9!$ (d) ${}^{10}P_6 \times {}^4P_3$

17. If x, y, z, are (m + 1) distinct prime numbers, the number of factors of $x^n yz...$is

(a) m(m + 1) (b) $2^n m$

(c) $(n + 1)2^m$ (d) $n2^m$

18. The number of ways in which one or more balls can be selected out of 10 white 9 green and 8 blue balls is

(a) 892 (b) 881

(c) 891 (d) 879

19. The number of all three elements subsets of the set $\{a_1, a_2, a_3, \dots a_n\}$ which contains a_3 is

(a) $^{n}C_3$ (b) $^{n-1}C_3$

(c) $^{n-1}C_2$ (d) none of these

20. The number of non negative integral solutions of $x + y + z \leq n$, where $n \in N$ is

(a) $^{n+3}C_3$ (b) $^{n+4}C_4$

(c) $^{n+5}C_5$ (d) none of these

21. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is

(a) $^{n-2}C_3$ (b) $^{n-3}C_2$

(c) $^{n-3}C_3$ (d) none of these

22. The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question is

(a) $^{21}C_7$ (b) $^{21}C_6$

(c) $^{21}C_8$ (d) none of these

23. If $^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then the value of r is

(a) 40 (b) 41

(c) 42 (d) none of these

24. If $^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the vale of n is

(a) 20 (b) 19

(c) 18 (d) 17

25. Ten different letter of an alphabet are given words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

(a) 33 (b) 44

(c) 48 (d) 52

26. A box contains two white balls, three lack balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw

(a) 129 (b) 84

(c) 64 (d) none of these

27. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary then the rank of the word RANDOM is

(a) 614 (b) 615

(c) 613 (d) 616

28. If the letters of the word MOTHER are written in all possible orders and these words are written out as in a dictionary, then the rank of the word MOTHER is

(a) 240 (b) 261

(c) 308 (d) 309

29. m men and n women are to be seated in a row so that no two women sit together. If m > n, then the number of ways in which they can be seated is

(a) $\frac{n!n!}{(m+n)!}$ (b) $\frac{m!(m+1)!}{(m-n+1)!}$

(c) $\frac{m!n!}{(m-n+1)!}$ (d) none of these

30. A five digit number divisible by 3 is to be formed using numerals 0, 1, 2, 3, 4, and 5 without repitition. The total number of ways this can be done

(a) 216 (b) 240

(c) 3125 (d) 600

31. A committee of 5 is to be formed from 9 ladies and 8 men. If the committee commands a lady majority, then the number of ways this can be done

(a) 2352 (b) 1008

(c) 3360 (d) 3486

32. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and fourth one having just one card

(a) $\frac{52!}{(17!)^3}$ (b) $\frac{52!}{(17!)^3 3!}$

(c) $\frac{51!}{(17!)^3}$ (d) $\frac{51!}{(17!)^3 3!}$

33. m parallel lines in a plane are intersected y a set of n parallel lines. The total number of parallelograms so formed is

(a) $\frac{(m-1)(n-1)}{4}$ (b) $\frac{mn}{4}$

(c) $\frac{m(m-1)n(n-1)}{2}$ (d) $\frac{mn(m-1)(n-1)}{4}$

34. There are n straight lines in a plane, no two of which are parallel, and no three pass through the same point. Their points of intersection are joined. The number of fresh lines thus obtained as

(a) $\frac{n(n-1)(n-2)}{8}$ (b) $\frac{n(n-1)(n-2)(n-3)}{6}$

(c) $\frac{n(n-1)(n-2)(n-3)}{8}$ (d) none of these

35. The straight lines I_1, I_2, I_3 are parallel and lie in the same plane. A total number of m points are taken on I_1; n points on I_2;k points on I_3. The maximum number of triangles formed with vertices at these points are

(a) ${}^{m+n+k}C_3$ (b) ${}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$

(c) ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$ (d) none of these

36. The number of ways in which we can distribute mn students equally among m sections is given by

(a) $\frac{(mn)!}{n!}$ (b) $\frac{(mn)!}{(n!)^m}$

(c) $\frac{(mn)!}{m!n!}$ (d) $(mn)^m$

37. If a polygon has 54 diagonals, the number of its sides is given by

(a) 12 (b) 11

(c) 10 (d) 9

38. The number of ways in which we can arrange 4 letters of the word MATHEMATICS is given by

(a) 136 (b) 2454

(c) 1680 (d) 192

39. Each of the five questions in a multiple-choice test has 4 possible answers. The number of different sets of possible answers is

(a) $4^5 - 4$ (b) $5^4 - 5$

(c) 1024 (d) 624

40. The number of squares which we can form on a chessboard is

(a) 64 (b) 160

(c) 224 (d) 204

Binomial Theorem

1. If n > 3, then

   xyz C_0 + (x – 1) (y – 1) (z – 1) C_1 + (x – 2) (y – 2) (z – 2) C_2 + ... + ... (x – n) (y – n) (z – n) C_n equals

   (a) xyz (b) nxyz

   (c) –xyz (d) none of these

2. The total number of dissimilar terms in the expansion of (x1 + x2 + ... + xn)3 is

   (a) n^3 (b) $\frac{n^3+3n^2}{4}$

   (c) $\frac{n(n+1)(n+2)}{6}$ (d) $\frac{n^2(n+1)^2}{4}$

3. The coefficient of x6 in the expansion of $(1+x^2-x^3)^8$ is

   (a) 80 (b) 84

   (c) 88 (d) 92

4. The digit at units place in the number $17^{1995} + 11^{1995} - 7^{1995}$ is

   (a) 0 (b) 1

   (c) 2 (d) 3

5. If n is an odd natural number, then $\sum_{r=0}^{n}\frac{(-1)^r}{{}^nC_r}$ equals

   (a) 0 (b) 1/n

   (c) $\frac{n}{2^n}$ (d) none of these

6. If the second, third, and fourth term in the expansion of (x + a)n are 240, 720, 1080 respectively, then the value of n is

   (a) 15 (b) 20

   (c) 10 (d) 5

7. If $n \in N$ such that $(7+4\sqrt{3})^n = 1+F$ where $I \in N$ and 0 < F < 1, then the value of (1 + F) (1 – F) is

   (a) an even integer (b) an odd integer

   (c) depends upon n (d) none of these

8. The sum of coefficients of the polynomial (1 + x – 3x2)2143

   (a) 0 (b) 1

   (c) 72n (d) 22n

9. If n is an even positive integer, then the condition taht the greatest term in the expansion of (1 + x)n may have the greatest coefficient also is

(a) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (b) $\frac{n+1}{n} < x < \frac{n}{n+1}$

(c) $\frac{n}{n+4} < x < \frac{n+4}{4}$ (d) none of these

10. If the r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ coefficient of $(1 + x)^n$ are in AP, then n is a root of the equation

(a) $x^2 - x(4r + 1) + 4r^2 - 2 = 0$ (b) $x^2 + x(4r + 1) + 4r^2 - 2 = 0$

(c) $x^2 + x (4r + 1) + 4r^2 + 2 = 0$ (d) none of these

11. If $C_0, C_1, C_2, ..., C_n$ are binomial coefficients in the expansion of $(1 + x)^n$, then the value of $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} .....(-1)^n \frac{C_n}{n+1}$

(a) 0 (b) $\frac{1}{n+1}$

(c) $\frac{2^n}{n+1}$ (d) $-\frac{1}{n+1}$

12. If n is an integer greater than unity, they the value of

$a - {}^nC_1(a - 1) + {}^nC_2 (a - 2) - {}^nC_2 (a - 3) + ... + (-1)^n (a - n)$ is

(a) 0 (b) 1

(c) n (d) −1

13. If C_r stands for ${}^nC_r$, then the sum of the series

$$a - \frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 + ... + (-1)^n (n+1)C_n^2]$$

(a) 0 (b) $(-1)^{\frac{n}{2}}(n+1)$

(c) $(-1)^{\frac{n}{2}}(n+2)$ (d) $(-1)^{\frac{n}{2}}n$

14. If the rth term in the expansion of $\left(\frac{x}{3} - \frac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to

(a) 2 (b) 3

(c) 4 (d) 5

15. The value of $C_0^2 + 3C_1^2 + 5C_2^2 + ...$ to (n + 1) terms, is

(a) ${}^{2n-1}C_{n-1}$ (b) $(2n + 1)\ {}^{2n-1}C_n$

(c) $2(n + 1)\ {}^{2n-1}C_{n-1}$ (d) ${}^{2n-1}C_n + (2n+1)\ {}^{2n-1}C_{n-1}$

16. The number of terms in the expansion of $(x + y + z)^{10}$ is

(a) 11 (b) 33

(c) 66 (d) 1000

17. If x is so small that its two and higher powers can be neglected and if

$(1-2x)^{-\frac{1}{2}}(1-4x)^{-\frac{5}{2}} = 1+kx$, then k is equal to

(a) 1 (b) −2

(c) 10 (d) 11

18. The number of terms independent of x in $\left(1+x+\frac{b}{x}\right)^{10}$ are

(a) 5 (b) 6

(c) 7 (d) 8

19. The coefficient of x^8 in the expansion of $(1+x)^{10}(1-2x+x^2)$ is

(a) ${}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8$ (b) ${}^{10}C_6 - {}^{10}C_7 + {}^{10}C_8$

(c) ${}^{10}C_6 - 2\,{}^{10}C_7 + {}^{10}C_8$ (d) ${}^{10}C_6 + 2\,{}^{10}C_7 + {}^{10}C_8$

20. The sum of the coefficient in the expansion of $(1 - 2x - 3x^2)^{20}$ is

(a) 2^{40} (b) 3^{20}

(c) 18763 (d) 10840

21. The sum of the series $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2$ is

(a) $\frac{2^n(1.3.5.7...(2n-1))}{n!}$ (b) $\frac{2^n(2.4.6...2n)}{n!}$

(c) $C(2n,\ n+1)$ (d) $\frac{2^n(2.4.6...2n)}{n!}$

22. The number of zero at the end of 1271 is

(a) 29 (b) 30

(c) 31 (d) 32

MATHEMATICS

SECTION - A

1. Find the ratio in which the line segment joining the points (1, 2) and (–2, 1) is divided by the line $3x + 4y = 7$.

2. A rod of length 1 slides with its ends on two perpendicular lines. Find the locus of the mid-point.

3. Prove that the line $5x - 2y - 1 = 0$ is mid parallel to the lines $5x - 2y - 9 = 0$ and
$5x - 2y + 7 = 0$.

4. Show that the area of the triangle with vertices at $(p - 4, p + 5)$, $(p + 3, p - 2)$ and
(p, p) is independent of p.

5. Find the circumcentre of the triangle whose sides are $3x - y + 3 = 0$, $3x + 4y + 3 = 0$ and $x + 3y + 11 = 0$.

6. A variable line passes through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ cuts the coordinate axes its A and B respectively. Find the locus of the mid-point AB

7. Prove that the quadrilateral whose vertices are A(–2, 5), B(4, –1), C(9, 1) and D(3, 7) is parallelogram, and find its area. Find the coordinates of a point E on AC such that it divides AC in the ratio 2 : 1. Prove that D, E and F, the mid point of BC are collinear.

8. Find the line which is parallel to x-axis and crosses thr curve $y=\sqrt{x}$ at an angle of 45^o.

9. Show that the straight lines $7x - 2y + 10 = 0$, $7x + 2y - 10 = 0$ and $y + 2 = 0$ form an isosceles triangle, and find its area.

10. Vertices of a ΔABC are A(2, 3), B(–4, –4), C(5, –8). Then find the length of the median through C.

11. If p be the length of perpendicular pass the origin on the line $\frac{x}{a}+\frac{y}{b}=1$ then $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

12. If P(1, 0), Q(–1, 0) and R(2, 0) are three given points, then find the locus of point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$.

13. Determine all values of α for which the point (α, α^2) lies inside the triangle formed by lines $2x + 3y - 1 = 0$, $x + 2y - 3 = 0$ and $5x - 6y - 1 = 0$.

14. If the line $3x + 4y - 24 = 0$ meets the coordinate axes at A and B, then find the incenter of the ΔOAB.

15. The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$ and $C(1, 4)$. Find the equation of the bisector of the angle $\angle ABC$

16. A line through $A(-5,-4)$ meets the line $x + 3y + 2 = 0, 2x + y + 4 = 0$ and $x - y - 5 = 0$ at the points B, C and D respectively. If $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$, find the equation of the line.

17. A rectangle PQRS has its side PQ parallel to the line $y = mx$ and vertices P, Q, and S on the lines $y = a$, $x = b$ and $x = -b$ respectively. Find the locus of vertex R.

18. Let $O(0, 0)$, $A(2, 0)$ and $B\left(1, \frac{1}{\sqrt{3}}\right)$ be the vertices of a triangle. Let R be the region consisting of all those points P inside ΔOAB which satisfy

$$d(P, OA) \leq \min\left[d(P, OB), d(P, AB)\right]$$

where d denotes the distance from the point to the corresponding line. Sketch the region R and find its area.

19. Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent.

20. Show that the centroid 'G' of a triangle divides the join of its orthocentre H and circumcentre S in the ratio 2 : 1.

SECTION - B

1. Find the ratio in which the line segment joining the points (1, 2) and (−2, 1) is divided by the line $3x + 4y = 7$.

(a) (3, 7) (b) (2, 4)

(c) (7, −2) (d) none of these

2. The triangle joining the points (2, 7), (4, −1), (2, 6) is

(a) equilateral (b) right angled

(c) Isoceles (d) scalene

3. The co-ordinates of the foot of the perpendicular from the point (2, 4) on the line $x + y = 1$ are

(a) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (b) $\left(\frac{-1}{2}, \frac{3}{2}\right)$

(c) $\left(\frac{4}{3}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{4}, \frac{-1}{2}\right)$

4. The points (2a, a), (a, 2a) and (a, a) enclose a triangle of area 8 units if

(a) $a = -4$ (b) $a = 4$

(c) $a = 2\sqrt{2}$ (d) none of these

5. The straight lines $x + y = 0$, $3x + y - 4 = 0$, $x + 3y - 4 = 0$ from a triangle which is

(a) equilateral (b) right angled

(c) isoceles (d) none of these

6. Every line of the system $(1 + 2\lambda)x + (\lambda - 1) y + 3 = 0$, λ being a parameter passes through a fixed point A. The equation of the line through A and parallel to the line $3x - y = 0$ is

(a) $3x - y + 5 = 0$ (b) $-3x + y + 5 = 0$

(c) $3x - y + 6 = 0$ (d) $3x - y + 8 = 0$

7. The lines $3x^2 - y^2 = 0$ and $x = 4$ enclose a triangle which is

(a) right angled (b) equilateral

(c) isoceles (d) none of these

8. If the lines $x + ay + a = 0$, $bx + y + b = 0$ and $cx + cy + 1 = 0$ (a, b, c being distinct ≠ 1) are concurrent, the value of $\frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}$ is

(a) 1 (b) 7

(c) 10 (d) 13

9. If the distance of any point (x, y) from the origin is defined as (x, y) = max {|x|, |y|} d(x, y) = a non-zero constant, then locus is

(a) circle (b) straight line

(c) square (d) none of these

10. The reflection of the point (4, −13) in the line $5x + y + 6 = 0$ is

(a) (−1, −14) (b) (3, 4)

(c) (1, 2) (d) (−4, 15)

11. The acute angle θ through which the coordinate axis should be rotated for the point A(2, 4) to attain the new abscissa 4 is given by

(a) $\tan\theta = \frac{3}{4}$ (b) $\tan\theta = \frac{5}{6}$

(c) $\tan\theta = \frac{7}{8}$ (d) none of these

12. If one of the diagonal of the square is along x = 2y and one of its vertices is (3, 0), then its, sides through the vertex is given by the equations

(a) $y - 3x + 3 = 0$, $3y + x + 9 = 0$ (b) $y - 3x + 9 = 0$, $3y - x + 3 = 0$

(c) $y - 3x + 9 = 0$, $3y + x - 3 = 0$ (d) $x + 3x - 3 = 0$, $3y + x + 9 = 0$

13. The bisector of the acute angle formed between the lines $4x - 3y + 7 = 0$ and $3x - 4y + 14 = 0$ has the equations

(a) $x + y - 7 = 0$ (b) $x - y + 3 = 0$

(c) $3x + y - 11 = 0$ (d) none of these

14. Let the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y, then

(a) the lines will p on through the fixed point

(b) there will be a set of parallel lines

(c) all the lines will intersect the line $x = x_1$

(d) none of these

15. The coordinates of the point P on the line $2x + 3y + 1 = 0$ such that $|PA - PB|$ is maximum where A is (2, 0) and B is (0, 2) is

(a) (7, −5) (b) (7, 6)

(c) (0, −5) (d) (0, 5)

SECTION - C

1. The orthocenter of the triangle formed by the line $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in

(a) I Quad. (b) II Quad

(c) III Quad. (d) IV Quad

2. The image of the point A(1, 2) be the line mirror $y = x$ is the point B and image of B by the line mirror $y = 0$ is the point (α, β) then

(a) $\alpha = 1, \beta = -2$ (b) $\alpha = 0, \beta = 0$

(c) $\alpha = 2, \beta = -1$ (d) $\alpha = 2, \beta = 2$

3. The equation of the line which passes through $(a\cos^3\theta, a \sin^3\theta)$ and perpendicular to the line $x \sec\theta + y \operatorname{cosec}\theta = a$ is

(a) $x \sin\theta + y \cos\theta = 2a \cos2\theta$ (b) $x \cos\theta + y \sin\theta = 2a \cos2\theta$

(c) $x \sin\theta + y \cos\theta = 2 \sin2\theta$ (d) $x \cos\theta - y \sin\theta = a \cos2\theta$

4. The equation of the straight line which passes through the point (1, −2) and cuts
x equal intercepts from axes will be

(a) $x + y = 1$ (b) $x - y = 1$

(c) $x + y + 1 = 0$ (d) none of these

5. The orthocentre of the triangle whose vertices are (0, 0), (3, 0) and (0, 4) is

(a) $\left(\frac{3}{2}, 2\right)$ (b) (0, 0)

(c) $\left(7, \frac{4}{3}\right)$ (d) (4, −4)

6. The distance between the lines 4x + 3y = 11 and 8x + 6y = 15 is

(a) 4 (b) 7/2

(c) 4/10 (d) 7/15

7. A point equidistant from the lines 4x + 3y + 10 = 0, 5x − 12y + 26 = 0 and 7x + 24y − 50 = 0 is

(a) (0, 0) (b) (0, 1)

(c) (1, −1) (d) (1, 1)

8. The lines 2x + y − 1 = 0, ax + 3y − 3 = 0 and 3x + 2y − 2 = 0 are concurrent for

(a) a = 0 only (b) all value of a

(c) $-1 \le a \le 3$ (d) a > 0 only

9. The straight line passes through the point (1, 1) and the portion of the lines intercepted between x and y axis is divided at the point in the ratio 3 :4. The equation of the line is

(a) 4x + 3y = 7 (b) 3x + 4y = 7

(c) 2x − 3y = 1 (d) 3x − 4y + 1 = 0

10. Area of the rhombus enclosed by the lines $ax \pm by \pm c = 0$ is

(a) $2c^2/ab$ (b) $2b^2/ca$

(c) $2a^2/bc$ (d) none of these

11. If a line making an angle θ with the positive direction of x axis, and is drawn through the point (2, 1) to intersect the line y − 2x + 6 = 0 at a distance $3\sqrt{2}$ from the point, then tanθ is equal to

(a) $\frac{1}{7}$ (b) 1

(c) 8 (d) 7

12. If A and B are two points having coordinate (3, 4) and (5, −2) respectively and P is a point such that PA = PB and area of triangle PAB = 10 sq.units then coordinates of P are

(a) (7, 2) or (1, 0) (b) (7, 4) or (13, 2)

(c) (4, 13) or (2, 2) (d) none of these

13. The equation of the locus of the point which moves so that the sum of the distance from two given points (ae, 0) and (–ae, 0) is equal to 2a, is

(a) $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ (b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

(c) $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ (d) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

14. The incentre of the triangle formed by the lines x = 0, y = 0 and 3x + 4y = 12 is

(a) (1, 1) (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, 1\right)$ (d) $\left(1, \frac{1}{2}\right)$

15. The value of λ for which the following three lines $x + y - 1 = 0$, $\lambda x + 2y - 3 = 0$,
$\lambda^2 x + 4y + 9 = 0$ are concurrent are

(a) 2, –15 (b) 3, 14

(c) 2, 15 (d) 6, 17

ANSWER

SECTION – A

1. 4 : 9

2. $4x^2 + 4y^2 = 1$

5. $3, -3$

6. $ab(x + y) = 2xy(a + b)$

8. $x = \frac{1}{2}$

9. 14 sq. units

10. $\sqrt{85}$

12. Straight lie parallel to y-axis

13. $\frac{-3}{2} < \alpha < -1$

14. (2, 2)

15. $7y = x + 2$

16. $2x + 3y + 22 = 0$

17. $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$

18. $(2 - \sqrt{3})$ sq.units

OBJECTIVE ANSWER

SECTION – A

1. (c)	2. (b)	3. (b)	4. (a)	5. (c)
6. (a)	7. (c)	8. (a)	9. (b)	10. (a)
11. (a)	12. (c)	13. (b)	14. (b, c)	15. (a)

SECTION – B

1. (a)	2. (c)	3. (d)	4. (c)	5. (b)
6. (c)	7. (a)	8. (b)	9. (a)	10. (a)
11. (b)	12. (a)	13. (d)	14. (a)	15. (a)

CIRCLES

1. If $y = 3x + c$ is a tangent to the circle $x^2 + y^2 - 2x - 4y - 5 = 0$, then c is

(a) 8 (b) 9 (c) 10 (d) 5

2. The points (-5, 11), (11, 19), (18, -4) lie in a circle. Centre of the circle is at

(a) (3, 4) (b) (4, 3) (c) (5, 4) (d) none of these

3. The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ touch each other.

The equation of their common tangent is

(a) $x = 3$ (b) $y = 6$ (c) $7x - 12y - 21 = 0$ (d) $7x + 12y + 21 = 0$

4. The length of the tangent from the point (5, 4) to the circle $x^2 + y^2 + 2x - 6y = 6$ is

(a) $\sqrt{21}$ (b) $\sqrt{38}$ (c) $2\sqrt{2}$ (d) $2\sqrt{13}$

5. The area of circle centred at (1, 2) and passing through (4, 6) is

(a) 5π (b) 10π (c) 25π (d) none of these

6. The equation of the circle passing through the intersection of $x^2 + y^2 + 13x - 13y = 0$ and

$2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and whose centre lies on $13x + 30y = 0$

(a) $x^2 + y^2 - 13x - 13y - 25 = 0$ (b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

(c) $2x^2 + 2y^2 + 30x - 13y - 25 = 0$ (d) $x^2 + y^2 + 30x - 13y + 25 = 0$

7. The locus of the centre of a circle radius 2 which rolls on the out side the circle

$x^2 + y^2 + 3x - 6y - 9 = 0$ is

(a) $x^2 + y^2 + 3x - 6y + 5 = 0$ (b) $x^2 + y^2 + 3x - 6y + 31 = 0$

(c) $x^2 + y^2 + 3x - 6y + 29/4 = 0$ (d) none of these

8. The distance between the chords of contact of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is

(a) $g^2 + f^2$ (b) $\frac{1}{2}\left(g^2 + f^2 + c^2\right)$

(c) $\frac{1}{2}\frac{g^2 + f^2 + c^2}{\sqrt{g^2 + f^2}}$ (d) $\frac{1}{2}\frac{g^2 + f^2 - c}{\sqrt{g^2 + f^2}}$

9.. The equation of circle with origin as centre and passing through the vertices of an equilateral

triangle whose median is of length 3a is

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$

10. A circle having area = 154 sq. units has two diameters $2x - 3y - 5 = 0$ and $3x - 4y - 7 = 0$, then the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y - 62 = 0$ (b) $x^2 + y^2 - 2x + 2y - 47 = 0$

(c) $x^2 + y^2 + 2x - 2y - 47 = 0$ (d) $x^2 + y^2 + 2x - 2y - 62 = 0$

11. The number of common tangents to the circle $x^2 + y^2 = 1$ and $x^2 + y^2 - 4x + 3 = 0$

(a) 1 (b) 2 (c) 3 (d) 4

12. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally then k equals

(a) 2 or -3/2 (b) -2 or -3/2 (c) 2 or 3/2 (d) -2 or 3/2

13. Four distinct points (2k, 3k) (1, 0)(0, 1) and (0, 0) lie on a circle for

(a) for all integral k (b) $k < 0$ (c) $0 < k < 1$ (d) for two values of k

14. The equation of the circle concentric to the circle $2x^2 + 2y^2 - 3x + 6y + 2 = 0$ and having area double the area of this circle is:

(a) $8x^2 + 8y^2 - 24x + 48y - 13 = 0$ (b) $16x^2 + 16y^2 + 24x - 48y - 13 = 0$

(c) $16x^2 + 16y^2 - 24x + 48y - 13 = 0$ (d) $8x^2 + 8y^2 + 24x - 48y - 13 = 0$

15. One of the diameter of the circle $x^2 + y^2 - 12x + 4y + 6 = 0$ is given by

(a) $x + y = 0$ (b) $x + 3y = 0$ (c) $x = y$ (d) $3x + 2y = 0$

16. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle drawn on this chord as diameter is:

(a) $x^2 + y^2 + ax = 0$ (b) $x^2 + y^2 - ax = 0$ (c) $x^2 + y^2 + ay = 0$ (d) $x^2 + y^2 - ay = 0$

17. The value of k so that $x^2 + y^2 + kx + 4y + 2 = 0$ and $2(x^2 + y^2) - 4x - 3y + k = 0$ cut orthogonally, is

(a) 10/3 (b) - 8/3 (c) - 10/3 (d) 8/3

18. The length of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is

(a) 9/2 (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\frac{3}{2}$

19. The limiting points of the system of circles represented by the equation $2\left(x^2 + y^2\right) + \lambda x + \frac{9}{2} = 0$, are

(a) $\left(\pm\frac{3}{2}, 0\right)$ (b) $(0,0)$ *and* $\left(\frac{9}{2}, 0\right)$ (c) $\left(\pm\frac{9}{2}, 0\right)$ (d) $(\pm 3, 0)$

20. For the given circles $x^2+y^2-6x-2y+1=0$ and $x^2+y^2+2x-8y+13=0$ which of the following is true ?

(a) One circle lies inside the other (b) One circle lies completely outside the other
(c) Two circles intersect in two points (d) They touch each other externally.

21. If the chord y = mx + 1 subtends an angle of measure 45° at the major segment of the circle $x^2 + y^2 = 1$ then value of m is

(a) $1\pm\sqrt{2}$ (b) $-2\pm\sqrt{2}$ (c) $-1\pm\sqrt{2}$ (d) ±1

22. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is

(a) $(y - q)^2 = 4px$ (b) $(y - q)^2 = 4py$
(c) $(y - p)^2 = 4qx$ (d) $(y - p)^2 = 4qy$

23. If the lines 2x + 3y + 1 = 0 and 3x – y – 4 = 0 lie along diameters of a circle of circumference 10π, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 23 = 0$ (b) $x^2 + y^2 - 2x - 2y - 23 = 0$
(c) $x^2 + y^2 + 2x + 2y - 23 = 0$ (d) $x^2 + y^2 - 2x + 2y - 23 = 0$

24. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is

(a) $x^2 + y^2 + x - y = 0$ (b) $x^2 + y^2 - x + y = 0$
(c) $x^2 + y^2 + x + y = 0$ (d) $x^2 + y^2 - x - y = 0$

25. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line 5x + by – a = 0 passes through P and Q for

(a) exactly one value of a (b) no value of a
(c) infinitely many values of a (d) exactly two values of a

26. A circle touches the x-axis and also touches the circle with center at (0, 3) and radius 2. The locus of the center of the circle is

(a) an ellipse (b) a circle (c) a hyperbola (d) a parabola.

27. If circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its center is

(a) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$ (b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(c) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$ (d) $2ax + 2by - (a^2 + b^2 + k^2) = 0$

28. If the pair of the lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of these sectors is thrice the area of another sector then

(a) $3a^2 - 10ab + 3b^2 = 0$ (b) $3a^2 - 2ab + 3b^2 = 0$

(c) $3a^2 + 10ab + 3b^2 = 0$ (d) $3a^2 + 2ab + 3b^2 = 0$

29. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y - 62 = 0$ (b) $x^2 + y^2 - 2x + 2y - 47 = 0$

(c) $x^2 + y^2 + 2x - 2y - 47 = 0$ (d) $x^2 + y^2 + 2x - 2y - 62 = 0$

30. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is

(a) $x^2 + y^2 = \frac{27}{4}$ (b) $x^2 + y^2 = \frac{9}{4}$

(c) $x^2 + y^2 = \frac{3}{2}$ (d) $x^2 + y^2 = 1$

31. Consider a family of circles which are passing through the point (-1, 1) and are tangent to x- axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval.

(a) $k \geq ½$ (b) $-1/2 \leq k \leq ½$ (c) $k \leq ½$ (d) $0 < k < ½$

32. The point diametrically opposite to the point P(1, 0)on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(a) (- 3, - 4) (b) (3, 4) (c) (3, -4) (d) (-3, 4)

33. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the co-ordinate axes, then one vertex of the square is

(a) $(1+\sqrt{2},-2)$ (b) $(1-\sqrt{2},-2)$ (c) $(1,-2+\sqrt{2})$ (d) none of these

34. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is

(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$

(c) $x^2 + y^2 - 4y + 2 = 0$ (c) none of these

35. If A and B are points in the plane such that $\frac{PA}{PB} = k$ (constant) for all P on a given circle, then we must have

(a) $k \in R - \{0, 1\}$ (b) $k \in R - \{1\}$ (c) $k \in R - \{0\}$ (d) none of these

36. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2

37. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is

(a) $\{(x, y):x^2 = 4y\} \cup \{(x, y):y \leq 0\}$ (b) $\{(x, y):x^2 + (y - 1)^2 = 4\} \cup \{(x, y):y \leq 0\}$

(c) $\{(x, y):x^2 = y\} \cup \{(0, y):y \leq 0\}$ (d) $\{(x, y):x^2 = 4y\} \cup \{(0, y):y \leq 0\}$

38. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(a) 3 **(b)** 2 **(c)** 3/2 **(d)** 1

ANSWERS(Circle)

1.	b	**2.**	d	**3.**	a	**4.**	a	**5.**	c	**6.**	a
7.	b	**8.**	d	**9.**	c	**20.**	b	**11.**	c	**12.**	a
13.	d	**14.**	c	**15.**	b	**16.**	b	**17.**	c	**18.**	b
19.	a	**20**.	d	**21.**	d	**22.**	d	**23.**	d	**24.**	d
25.	b	26.	d	**27.**	d	28.	d	29.	b	**30.**	b
31.	c	**32.**	a	**33**.	d	**34.**	b	**35.**	a	**36.**	c
37.	d	**38**.	b								

CONICS

1. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if
(a) $m = 1$ (b) $m = 2$ (c) $m = 4$ (d) $m = 3$

2. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is
(a) (-1, 0) (b) (-1, -1) (c) (0, -1) (d) none of these

3. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 4$. Let P and Q be the points (1, 2) and (2, 1) respectively. Then
(a) Q lies inside C but outside E (b) Q lies outside both C and E
(c) P lies inside both C and E (d) P lies inside C but outside E

4. The foci of the ellipse $25(x + 1)^2 + 9(y + 2)^2 = 225$, are at
(a) (-1, -2) and (-1, -6) (b) (-2, 1) and (-2, 6)
(c) (-1, 2) and (-1, 6) (d) (-1, -20 and (-2, -1)

5. The vertex of the parabola $y^2 = 4(a' - a)(x - a)$ is
(a) (a′, a) (b) (a, a′) (c) (a, 0) (d) (a′, 0)

6. If the line $y = 3x + \lambda$ touches the hyperbola $9x^2 - 5y^2 = 45$, then the value of λ is
(a) 36 (b) 45 (c) 6 (d) 15

7. If the parabola $y^2 = 4ax$ passes through (3, 2) then the length of its latus-rectum is
(a) 2/3 (b) 4/3 (c) 1/3 (d) 4

8. The eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is
(a) 1/3 (b) 2/3 (c) ¾ (d) none of these

9. the vertex of the parabola $x^2 + 8x + 12y - 8 = 0$ is
(a) (-4, 2) (b) (4, -1) (c) (-4, -1) (d) (4, 1)

10. P is any point on the ellipse $81x^2 + 144y^2 = 1944$ whose foci are S and S′. Then SP + S′P equals
(a) 3 (b) $4\sqrt{6}$ (c) 36 (d) 324

11. What is the equation of the central ellipse with foci (± 2, 0) and eccentricity ½ ?

(a) $3x^2 + 4y^2 = 48$ (b) $4x^2 + 3y^2 = 48$ (c) $3x^2 + 4y^2 = 12$ (d) $4x^2 + 3y^2 = 12$

12. $x^2 - 4y^2 - 2x + 16y - 24 = 0$ represents
(a) straight lines (b) an ellipse (c) a hyperbola (d) a parabola

13. The line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if

(a) $\frac{a^2}{m^2}+\frac{b^2}{l^2}=\frac{\left(a^2-b^2\right)}{n^2}$ (b) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)}{n^2}$

(c) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)}{n^2}$ (d) none of these

14. If the straight line $y = 2x + c$ is a tangent to the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$, then c will be equal to
(a) ± 4 (b) ± 6 (c) ± 1 (d) ± 8

15. The curve represented $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$ is
(a) ellipse (b) parabola (c) hyperbola (d) circle

16. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then
(a) $4al + n = 0$ (b) $4al + 4am + n = 0$ (c) $4am + n = 0$ (d) $al + n = 0$

17. If the normal at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, meets the aces in G and g respectively, then PG : Pg
(a) a : b (b) $a^2 : b^2$ (c) $b^2 : a^2$ (d) b : a

18. The curve given by $x = \cos 2t$, $y = \sin t$
(a) ellipse (b) circle (c) part of parabola (d) hyperbola

19. Length of major axis of ellipse $9x^2 + 7y^2 = 63$ is
(a) 3 (b) 9 (c) 6 (d) $2\sqrt{7}$

20. The angle between the pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ from the point (1, 2) is

(a) $\tan^{-1}\left(\frac{12}{5}\right)$ (b) $\tan^{-1}\left(6\sqrt{5}\right)$ (c) $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$ (d) $\tan^{-1}\left(12\sqrt{5}\right)$

21. The number of parabolas that can be drawn if two ends of the latus rectum are given, is

(a) 1 (b) 2 (c) 4 (d) 5

22. Equation of tangent to hyperbola $\frac{x^2}{3}-\frac{y^2}{2}=1$ equally inclined to coordinate axis is

(a) y = x + 1 (b) y = x – 1 (c) y = x + 2 (d) y = x – 2

23. The locus of a point whose distance from a fixed point and the fixed straight line x = 9/2 is always in the ratio 2/3 is

(a) hyperbola (b) ellipse (c) parabola (d) circle

24. If e and e′ are the eccentricities of hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its conjugate hyperbola, then the value of $\frac{1}{e^2}+\frac{1}{e'^2}$ is:

(a) 0 (b) 1 (c) 2 (d) none of these

25. In an ellipse the angle between the lines joining the foci with the positive end of minor axis is a right angle, the eccentricity of the ellipse is:

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

26. The equation of tangent to the curve $\frac{x^2}{3}-\frac{y^2}{2}=1$ which is parallel to y = x, is

(a) y = x ± 1 (b) y = x – ½ (c) y = x + ½ (d) y = 1 – x

27. The foci of the conic section $25x^2 + 16y^2 - 150x = 175$ are

(a) (0, ± 3) (b) (0, ± 2) (c) (3, ± 3) (d) (0, ± 1)

28. The product of perpendiculars drawn from any point of a hyperbola to its asymptotes is

(a) $\frac{a^2b^2}{a^2+b^2}$ (b) $\frac{a^2+b^2}{a^2b^2}$ (c) $\frac{ab}{\sqrt{a}+\sqrt{b}}$ (d) $\frac{ab}{a^2+b^2}$

29. The equation of the ellipse whose distance between the foci is equal to 8 and distance between the directrix is 18, is

(a) $5x^2-9y^2=180$ (b) $9x^2+5y^2=180$ (c) $x^2+9y^2=180$ (d) $5x^2+9y^2=180$

30. The eccentricity of the hyperbola can never be equal to

(a) $\sqrt{\frac{9}{5}}$ (b) $2\sqrt{\frac{1}{9}}$ (c) $3\sqrt{\frac{1}{8}}$ (d) 2

31. If the line $y=2x+\lambda$ be a tangent to the hyperbola $36x^2-25y^2=3600$, then λ is equal

(a) 16 (b) -16 (c) ±16 (d) None of these

32. The equation $y^2-x^2+2x-1=0$ represents

(a) a hyperbola (b) an ellipse
(c) a pair of straight lines (d) a rectangular hyperbola

33. Two common tangents to the circle $x^2 + y^2 = a^2$ and parabola $y^2 = 8ax$ are

(a) x = ±(y + 2a) (b) y = ±(x + 2a) (c) x = ±(y + a) (d) y = ±(x + a)

34. The normal at $(bt_1^2, 2bt_1)$ on parabola $y^2 = 4bx$ meets the parabola again in point $(bt_2^2, 2bt_2)$, then

(a) $t_2=t_1+\frac{2}{t_1}$ (b) $t_2=-t_1-\frac{2}{t_1}$ (c) $t_2=-t_1+\frac{2}{t_1}$ (d) $t_2=t_1-\frac{2}{t_1}$

35. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is

(a) 9 (b) 1 (c) 5 (d) 7

36. If a ≠ 0 and the line 2bx + 3cy + 4d = 0 passes through the points of intersection of the parabola $y^2 = 4ax$ and $x^2 = 4ay$, then

(a) $d^2 + (3b - 2c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b - 3c)^2 = 0$ (d) $d^2 + (2b + 3c)^2 = 0$

37. The eccentricity of an ellipse, with its centre at the origin, is ½. If one of the directrices is x = 4, then the equation of the ellipse is

(a) $4x^2 + 3y^2 = 1$ (b) $3x^2 + 3y^2 = 12$ (c) $4x^2 + 3y^2 = 12$ (d) $3x^2 + 3y^2 = 1$

38. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x + 2 = 0$ (c) $x^2 - 4x + 2 = 0$ (d) $x^2 + 4y + 2 = 0$

39. The locus of a point P(α, β) moving under the condition that the line y = αx + β is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(a) an ellipse (b) a circle (c) a parabola (d) a hyperbola

40. The locus of the vertices of the family of parabolas $y = \frac{a^2x^2}{3} + \frac{a^2x}{2} - 2a$ is

(a) $xy = \frac{35}{16}$ (b) $xy = \frac{64}{105}$ (c) $xy = \frac{105}{64}$ (d) $xy = \frac{3}{4}$

41. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?

(a) Directrix (b) Abscissae of vertices (c) Abscissae of foci (d) Eccentricity

42. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

(a) $(0, 2)$ (b) $(2, 4)$ (c) $(-2, 0)$ (d) $(-1, 1)$

43*. The normal to a curve at P(x ,y) meets the x-axis at G .If the distance of G from the origin is twice the abscissa of P, then the curve is a

(a) parabola (b) circle (c) hyperbola (d) ellipse

44. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{8}{3}$ (d) $\frac{2}{3}$

45. A parabola has the origin as its focus and the line x = 2 as the directrix . Then the vertex of the parabola is at

(a) $(0, 1)$ (b) $(2, 0)$ (c) $(0, 2)$ (d) $(1, 0)$

46. A hyperbola, having the transverse axis of length 2sin θ, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

(a) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ **(b)** $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

(c) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ **(d)** $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

ANSWERS(Conics)

1.	a	**2.**	a	**3.**	d	**4.**	c	**5.**	c	**6.**	c
7.	b	**8.**	b	**9.**	a	**10.**	b	**11.**	a	**12.**	c
13.	b	**14.**	b	**15.**	a	**16.**	a	**17.**	c	**18.**	c
19.	c	**20.**	c	**21.**	b	**22.**	a	**23.**	b	**24.**	b
25.	a	**26.**	a	**27.**	c	28.	a	**29.**	d	**30.**	b
31.	c	**32.**	c	**33.**	b	**34.**	b	**35.**	d	**36.**	d
37.	b	**38.**	a	**39.**	d	**40.**	c	**41.**	c	**42.**	c
43.	c	**44**.	c	**45.**	d	**46.**	a				

Function

1. The function f: N → N (N is the set of natural numbers) defined by f(n) = 2n + 3 is

(a) surjective (b) not surjective
(c) not injective (D) none of these

2. If $e^x = y + \sqrt{1+y^2}$, then y is equal to

(a) $e^x + e^{-x}$ (b) $e^x - e^{-x}$ (c) $\frac{e^x - e^{-x}}{2}$ (d) $\frac{e^x + e^{-x}}{2}$

3.** The equation sin x - π/2 + 1 = 0 has one root in interval

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \pi\right)$ (c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) none of these

4. If f(x) = $\log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then f[g(x)] equals

(a) -f(x) (b) 3f(x) (c) $(f(x))^3$ (d) -3f(x)

5. The domain of the function $f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is

(a) [1, 4] (b) (0, 5) (c) (1, 5) (d) (0, 4)

6.. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which f(x) = 4^{-x^2} + $\cos^{-1}\left(\frac{x}{2}-1\right) + \log(\cos x)$ is defined is

(a) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (b) [0, π] (c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

7. The value of n ∈ I, for which the period of the function $f(x) = \frac{\sin nx}{\sin x/n}$ is 4π,is

(a) -3 (b) 3 (c) 2 (d) 4

8. The function $f(x) = \log\frac{1+x}{1-x}$ satisfy the equation

(a) $f(x_1)f(x_2) = f(x_1 + x_2)$ (b) $f(x+2) - 2f(x+1) + f(x) = 0$

(c) $f(x)+f(x+1)=f(x^2+x)$ (d) $f(x_1)+f(x_2)=f\left(\frac{x_1+x_2}{1+x_1x_2}\right)$

9. Let $f:R\to R$ be a function defined by $f(x)=\frac{x-m}{x-n}$, where $m\neq n$. Then

(a) f is one-one onto (b) f is one-one into

(c) f is many one onto (d) f is many one into

10. $f(x)=\frac{1}{x^2}$ the function is

(a) symmetric about x-axis (b) symmetric about y-axis

(c) symmetric about x and y-axis (d) none of these.

11. If $f(-x)=-f(x),\ then\ \ f(x)\ \ is$

(a) an even function (b) an odd function

(c) neither odd nor even (d) periodic function

12. The function $f(x)=x-[x]$, ([] represent greatest integer function), is

(a) a constant function (b) periodic with period ½

(c) periodic with period 1 (d) none periodic function

13. If $g:[-2,2]\to R$ where g(x) = $x^3+\tan x+\left[\frac{x^2+1}{p}\right]$ (where [x] denotes greatest integer ≤ x) be an odd function, then the value of the parameter p is

(a) -5 < p < 5 (b) p < 5 (c) p > 5 (d) none of these.

14. Let g(x) = 1 + x – [x] and $f(x)=\begin{cases}-1 & if \quad x<0\\ 0 & if \quad x=0\\ 1 & if \quad x>0\end{cases}$ then ∀x, fog(x) equals

(a) x (b) 1 (c) f(x) (d) g(x)

15. The domain of the function $f(x)=\frac{\cos^{-1}x}{[x]}$ is:

(a) [-1,0) ∪ {1} (b) [-1, 1] (c) [-1, 1) (d) None of these

16. If ϕ(x) is the inverse of the function f(x) and $f'(x)=\frac{1}{1+x^5}$, then $\frac{d}{dx}\phi(x)$ is

(a) $\frac{1}{1+\{\phi(x)\}^5}$ (b) $\frac{1}{1+\{f(x)\}^5}$ (c) $1+\{\phi(x)\}^5$ (d) 1 + f(x)

17. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(a)

[1,9] (b) [- 1, 9] (c) $[-9,1]$ (d) [-9, -1]

18. Let $f: N \to N$ defined by $f(x) = x^2 + x + 1, x \in N$, then f is

(a) One one onto (b) Many one onto

(c) One-one but not onto (d) None of the above

19. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(a) [1, 9] (b) [-1, 9] (c) [-9, 1] (d) [-9, -1]

20. The function $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$, is

(a) neither an even nor odd function (b) an even function

(c) an odd function (d) a periodic function

21. Domain of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}\left(x^3 - x\right)$, is

(a) (-1, 0) ∪ (1, 2) ∪ (2, ∞) (b) (a, 2)

(c) (-1, 0) ∪ (a, 2) (d) (1, 2) ∪ (2, ∞)

22. If f: R → R satisfies f(x + y) = f(x) + f(y), for all x, y ∈ R and f(1) = 7, then $\sum_{r=1}^{n} f(r)$, is

(a) $\frac{7n(n+1)}{2}$ (b) $\frac{7n}{2}$ (c) $\frac{7(n+1)}{2}$ (d) 7n + (n + 1)

23. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}.

The relation R is

(a) reflexive (b) transitive (c) not symmetric (d) a function

24. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

(a) {1, 2, 3} (b) {3, 4, 5, 6} (c) {1, 2, 3, 4} (d) {1,2,3,4,5}

25. Let f : (-1, 1) → B, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1 - x^2}$, then f is both one-one and onto when B is the interval

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

26. A function is matched below against an interval where it is supposed to be increasing

Which of the following pairs is incorrectly matched

	Interval	Function
(a)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(d)	$(-\infty, -4]$	$x^3 + 6x^2 + 6$

27. A real valued function f(x) satisfies the functional equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ where a is a given constant and $f(0) = 1, f(2a-x)$ is equal to

(a) -f(x) (b) f(x) (c) f(a) + f(a – x) (d) f(-x)

28. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function .

$\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$ is defined, is

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $[-\frac{\pi}{4}, \frac{\pi}{2})$ (c) $[0, \frac{\pi}{2})$ (d) $[0, \pi)$

29. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is

(a) $g(y) = \frac{y+3}{4}$ (b) $g(y) = \frac{y-3}{4}$ (c) $g(y) = \frac{3y+4}{3}$ (d) $g(y) = 4 + \frac{y+3}{4}$

30. Let R be the set of real numbers. If F : R → R is a function defined by $f(x) = x^2$ then f is

(a) injective but not surjective (b) surjective but not injective
(c) bijective (d) none of these.

31. If the values of the function f(x) = cos x – x(1 + x) $\left\{\frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$ lies between a and b then

(a) $a = \frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right),\ b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)$

(b) $a = \frac{1}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right),\ b = \frac{\sqrt{3}}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right)$

(c) $a = \frac{\sqrt{3}}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right),\ b = \frac{1}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)$

(d) none of these

32. $f(x)=\sqrt{\left(\frac{(x+1)(x-3)}{(x-2)}\right)}$ then the domain of f(x) must be

(a) $]-\infty,-1]\cup[3,\infty[$ (b) $]-\infty,-1]\cup[2,3]$

(c) $[-1,2[\cup[3,\infty[$ (d) none of these

33. Let f be an injective map with domain {x, y, z} and range {1, 2, 3} such that exactly one the three statements is correct and the remaining two are false f(x) = 1, f(y) ≠ 1, f(z) ≠ 2, then value of $f^{-1}(1)$ is

(a) x (b) y (c) z (d) none

34. If p, q, r are any real numbers, then

(a) max (p, q) < max (p, q, r) (b) min (p, q) = $\frac{1}{2}\left(q-p-|p-q|\right)$

(c) max (p, q) < min (p, q, r) (d) none of these.

35. If f(x) = cos(log x), then f(x) f(y) =

(a) ½ [f(y/x) + f(xy)] (b) ½ [f(x/y) + f(xy)]

(c) ½ [f(xy) + f(y)] (d) none

36. If the function f(x) and g(x) are defined on R → R such that

$f(x)=\begin{cases}0, & x\in \text{ rational}\\ x, & x\in \text{ irrational}\end{cases}$, $g(x)=\begin{cases}0, & x\in \text{ irrational}\\ x, & x\in \text{ rational}\end{cases}$, then (f – g) (x) is

(a) one-one and onto (b) neither one-one nor onto

(c) one-one but not onto (d) onto but not one-one

37. X and Y are two sets and f : X → Y, if {f(c) = y; c ⊂ X, y ⊂ Y}, $\{f^{-1}(d) = x, d \subset Y, x \subset X\}$, then the true statement is

(a) $f\left(f^{-1}(b)\right)=b$ (b) $f^{-1}\left(f(a)\right)=a$

(c) $f\left(f^{-1}(b)\right)=b, b\subset y$ (d) $f^{-1}\left(f(a)\right)=a, a\subset x$

Answers(Function)

1.	b	**2.**	c	**3.**	a, b	**4.**	b	**5.**	a	**6.**	c
7.	c	**8.**	d	**9.**	b	**10.**	b	**11.**	b	**12.**	c
13.	c	**14.**	b	**15.**	a	**16.**	c	**17.**	a	**18.**	a
19.	a	**20.**	c	**21.**	a	**22.**	a	**23.**	c	**24.**	a
25.	c	**26.**	c	**27.**	a	**28.**	c	**29.**	b	30.	d
31.	a	**32.**	c	**33.**	b	**34.**	b	**35.**	b	**36.**	a

37. d

Limit, Continuity and Differentiability

1. If $f(x)=\begin{cases} x \quad , & if \;\; x \; is \; rational \\ 1-x, & if \;\; x \; is \; irrational \end{cases}$, then

(a) f is only right continuous at x = ½ (b) f is only left continuous at x = ½

(c) f is continuous at x = ½ (d) f is discontinuous at all points.

2. $\lim_{x\to 0}\frac{1-\cos 8x}{1-\cos 6x}=$

(a) 64/36 (b) 5/8 (c) 64/25 (d) 15/23

3. If f(x) = x sin 1/x, x = 0, then value of function at x = 0, so that the function is continuous at x = 0, is

(a) 1 (b) 0 (c) -1 (d) indeterminate

4. In order that the function $f(x) = (x + 1)^{\cot x}$ is continuous at x = 0, f(x) must be defined as

(a) f(0) = 0 (b) f(0) = e (c) f(0) = 1/e (d) none of these

5. If f(x) and $\left|f(x)\right|$ are both differentiable, then f(x) is

(a) only left continuous at x = 0 (b) only right continuous at x = 0

(c) not continuous at x = 0 (d) continuous every where

6. Let $f(x)=\begin{cases}\frac{x}{2}-1, & 0\le x\le 1\\ \frac{1}{2}, & 1\le x\le 2\end{cases}$, g(x) = (2x + 1)(x – k) + 3, 0 ≤ x ≤ ∞, then g[f(x)] is continuous at x = 1, if k equals

(a) ½ (b) 11/6 (c) 1/6 (d) 13/6

7. Evaluate the $\lim_{n\to\infty}\left[\frac{n}{n^2+1}+\frac{n}{n^2+2^2}+.......+\frac{1}{2n}\right]$

(a) π (b) π/3 (c) π/2 (d) π/4

8. $\lim_{x\to e}\frac{\log x-1}{x-e}=$

(a) 1 (b) ½ (c) 1/e (d) 0

9. The value of f(0), so that the function $f(x)=\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$ is continuous at x = 0, is

(a) 2 (b) ¼ (c) 1/6 (d) 1/3

10. The set of all points where the function f(x) = x |x|, is differentiable is

(a) $(-\infty,0)\cup(0,\infty)$ (b) $(-\infty,0)$ (c) $(-\infty,\infty)$ (d) $(0,\infty)$

11. $\lim_{x\to\frac{\pi}{3}}\frac{\left[\sin\left(\pi/3-x\right)\right]}{(2\cos x-1)}$ is equal to

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) ½

12. The function $f(x)=\begin{cases}|2x-3|\cdot[x], & x\ge 1\\ \sin\left(\frac{\pi x}{2}\right), & x<1\end{cases}$

(where [x] denotes greatest integer ≤ x)

(a) is continuous at x = 2 (b) is differentiable at x = 1

(c) is continuous but not differentiable at x = 1 (d) none of these

13. The value of f(0), so that the function $f(x)=\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}(x\ne 0)$ is continuous is given by

(a) 2/3 (b) 6 (c) 2 (d) 4

14. If f(x), g(x) be twice differentiable functions on [0, 2], satisfying $f''(x)-g''(x), f'(1)=2, g(1)=4$ and f(2) = 3, g(2) = 9, then f(x) – g(x) at x = 4 equals

(a) 0 (b) -10 (c) 8 (d) 2

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15. $\lim_{n\to\infty}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+.......+\frac{1}{nb}\right]$ is equal to

(a) log(b/a) (b) log(a/b) (c) log a (d) log b

16. The value of $\lim_{x\to\infty}\frac{\sqrt{1+x^4}-\left(1+x^2\right)}{x^2}$ is equal to

(a) 0 (b) -1 (c) 2 (d) none of these

17. If 0 < x < y, then $\lim_{n\to\infty}\left(y^n+x^n\right)^{1/n}$ is equal to

(a) e (b) x (c) y (d) none of these

18. The function $f(x)=\begin{cases}x+2\ , & 1\le x\le 2\\ 4\ , & x=2\\ 3x-2, & x>2\end{cases}$ is continuous at

(a) x = 2 only (b) x ≤ 2 (c) x > 2 (d) none of these

19. $\lim_{x\to 9}\left[\frac{x^{3/2}-27}{x-9}\right]=$

(a) 3/2 (b) 9/2 (c) 2/3 (d) 1/3

20. $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{x^2}=$

(a) $\frac{11e}{24}$ (b) $\frac{-11e}{24}$ (c) $\frac{e}{24}$ (d) none of these [

21. $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}$

(a) sin 1 (b) 2 (c) π (d) 3

22. If f(x) = sin |x|, then f(x) is not differentiable at

(a) x = 0 only (b) all x

(c) multiples of π (d) multiples of π/2

23. $\lim_{n\to\infty}\left(1+\sin\frac{a}{n}\right)^n$ equals to

(a) e^a (b) e (c) e^{2a} (d) 0

24. Function f(x) = (|x - 1| + |x - 2| + cos x) where x ∈ [0, 4] is not continuous at number of points

(a) 1 (b) 2 (c) 3 (d) 0

25. $\lim_{n\to\infty}\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.......+\frac{1}{(2n-1).(2n+1)}$

(a) ½ (b) 1/3 (c) ¼ (d) none of these

26. The value of $\lim_{x\to 0}\dfrac{e^{ax}-e^{bx}}{x}$ equals

(a) a + b (b) a – b (c) e^{ab} (d) 1

27. Let $f(x)=\begin{cases} x^p \sin\dfrac{1}{x}, & x\neq 0 \\ 0 \quad , & x=0 \end{cases}$ then f(x) is continuous but not differential at x = 0 if

(a) $0 < p \le 1$ (b) $1 \le p < \infty$ (c) $-\infty < p < 0$ (d) p = 0

28. If $f(x)=\begin{cases} \dfrac{1-|x|}{1+x}, & x\neq -1 \\ 1 \quad , & x=-1 \end{cases}$ then f([2x]) (where [] represent greatest integer function),is

(a) continuous at x = -1 (b) continuous at x = 0

(c) discontinuous x = ½ (d) all of these

29. Let $f(x)=\begin{cases} \dfrac{\tan x-\cot x}{x-\frac{\pi}{4}}, & x\neq \dfrac{\pi}{4} \\ a \quad , & x=\dfrac{\pi}{4} \end{cases}$ the value of a so that f(x) is continuous at $x=\dfrac{\pi}{4}$ is:

(a) 2 (b) 4 (c) 3 (d) 1

30.**The value of $\lim_{\alpha\to 0}\dfrac{\cos ec^{-1}(\sec\alpha)+\cot^{-1}(\tan\alpha)+\cot^{-1}\cos(\sin^{-1}\alpha)}{\alpha}$

(a) is 0 (b) is -1 (c) is -2 (d) does not exist

31. Let g(x) = 1 + x – [x] and $f(x)=\begin{cases} -1, & x<0 \\ 0 \;, & x=0 \\ 1 \;, & x>0 \end{cases}$ then for all x, f[g(x)] is equal to

(a) x (b) 1 (c) f(x) (d) g(x)

32. $\lim_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is equal to

(a) 2a sin a (b) a^2 cos a

(c) a^2 cos a + 2a sin a (d) none of these.

33. If f(x) = $x\left(\sqrt{x}+\sqrt{1+x}\right)$, then

(a) f(x) is continuous but not differentiable at x = 0

(b) f(x) is differentiable at x = 0

(c) f(x) is not differentiable at x = 0 (d) none of these

34. The value of $\lim_{x\to 0}\dfrac{1+\sin x-\cos x+\log(1-x)}{x^3}$ is

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(a) -1 (b) ½ (c) -1/2 (d) 1

35. The value of $\lim_{x\to 0}\frac{\left(4^x-1\right)^3}{\sin\frac{x^2}{4}\log(1+3x)}$ is

(a) $\frac{4}{3}(\ln 4)^2$ (b) $\frac{4}{3}(\ln 4)^3$ (c) $\frac{3}{2}(\ln 4)^2$ (d) $\frac{3}{2}(\ln 4)^3$

36. If f(x) is differentiable function and f″(0) = a, then $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is equal to

(a) 3a (b) 2a (c) 5a (d) 4a

37. If function $f(x)=\begin{cases} x, & if\ x\ is\ rational \\ 1-x, & if\ x\ is\ irrational \end{cases}$ then the number of points at which f(x) is continuous, is

(a) ∞ (b) 1 (c) 0 (d) none of these

38. Let f(a) = g(a) = k and there nth derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}=4$ then the value of k is

(a) 0 (b) 1 (c) 2 (d) 4

39. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{1/x}$

(a) 0 (b) 4 (c) 2 (d) 1

40. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is

(a) ∞ (b) 1/8 (c) 0 (d) 1/32

41. Let $f(x)=\frac{1-\tan x}{4x-\pi}, x\neq\frac{\pi}{4}, x\in\left[0,\frac{\pi}{2}\right]$. If f(x) is continuous in $\left[0,\frac{\pi}{4}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(a) -1 (b) ½ (c) - ½ (d) 1

42. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are

(a) a = 1 and b = 2 (b) a = 1, b ∈ R
(c) a ∈ R, b = 2 (d) a ∈ R, b ∈ R

43. Suppose f(x) is differentiable at x = 1 and $\lim_{h\to 0}\frac{1}{h}(1+h)=5$, then f ′(1) equals

(a) 3 (b) 4 (c) 5 (d) 6

44. Let f be differentiable for all x. If f(1) = -2 and f ′(x) ≥ 2 for x ∈ [1, 6], then

(a) f(6) ≥ 8 (b) f(6) < 8

(c) f(6) < 5 (d) f(6) = 5

45. If f is a differentiable function satisfying $|f(x) - f(y)| \le (x - y)^2$, x, y ∈ R and f(0) = 0, then f(1) equals

(a) -1 (b) 0 (c) 2 (d) 1

46. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then

$\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0 (c) $\frac{-a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$

47. $\lim_{x \to \infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ equals

(a) ½ sec 1 (b) ½ cosec 1 (c) tan 1 (d) ½ tan 1

48. The set of points where $f(x) = \frac{x}{1 + |x|}$ is differentiable is

(a) (-∞, ∞) (b) (0, ∞) (c) (-∞, 0) ∪(0, ∞) (d) (-∞, -1)∪(-1, ∞)

49. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \text{Min}\ \{x + 1, |x| + 1\}$. Then which of the following is true ?

(a) $f(x)$ is not differentiable at x = 1 (b) $f(x)$ is differentiable every where .

(c) $f(x)$ is not differentiable at x = 0 (d) $f(x) \ge 1$ for all $x \in \mathbb{R}$

50. The function $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at x = 0 by defining $f(0)$ as

(a) -1 (b) 0 (c) 1 (d) 2

51. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(a) 12 (b) 4 (c) -4 (d) -12

52. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \ne 1 \\ 0 & \text{if } x = 1 \end{cases}$. Then which one of the following is true?

(a) f is differentiable at x = 0 but not at x = 1

(b) f is differentiable at x = 1 but not at x = 0

(c) f is neither differentiable at x = 0 nor at x = 1

(d) f is differentiable at x = 0 and n at x = 1

53. If $f(x)=\sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$, then $\lim\limits_{x\to\infty} f(x)$ is

(a) 0 (b) ∞ (c) 1 (d) none of these

54. For a real number y, let [y] denote the greatest integer less than or equal to y.

Then the function $f(x)=\dfrac{\tan(\pi[x-\pi])}{1+[x]^2}$ is

(a) discontinuous at some x

(b) continuous at all x, but the derivative f ′(x) does not exist for some x

(c) f ′(x) exists for all x, but the derivative f ″(x) does not exist for some x

(d) f ″(x) exist for all x

55. Let $f(x)=\begin{cases}(x-1)^2\sin\dfrac{1}{(x-1)}-|x|, & if\ x\neq 1\\ -1 &, if\ x=1\end{cases}$ be a real valued function. Then the set of points where f(x) is not differentiable is

(a) {0} (b) {0, 1} (c) {, 1, -1} (d) none of these

56. Let $f(x)=\begin{cases}\dfrac{(x^3+x^2+16x+20)}{(x-2)^2}-|x|, & if\ x\neq 2\\ k &, if\ x=2\end{cases}$. If f(x) is continuous for all x, then k =

(a) 7 (b) 2 (c) 0 (d) -1

57. If $\lim\limits_{x\to a}\left[f(x)g(x)\right]$ exists then

(a) both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exist

(b) both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ do not exist

(c) both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ may not exist (d) none of these

58. $\lim\limits_{x\to 0}\dfrac{2^x-1}{(1+x)^{\frac{1}{2}}-1}=$

(a) log 4 (b) log 2 (c) 1 (d) 0

59. Let $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)}; 0<x<2$, m and n are integers , $m\neq 0, n>0$, and let p be the left hand derivative of | x – 1| at x = 1. If $\lim\limits_{x\to 1^+} g(x)=p$, then

(a) $n=1, m=1$ (b) $n=1, m=-1$ (c) $n=2, m=2$ (d) $n>2, m=n$

60. Let f(x) be a non-constant twice differentiable function defined on $(-\infty,\infty)$ such that $f(x)=f(1-x)$ and $f'\left(\frac{1}{4}\right)=0$. Then ,

(a) $f''(x)$ vanishes at least twice on [0 , 1] (b) $f'\left(\frac{1}{2}\right)=0$

(c) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x dx=0$ (d)
$\int_{0}^{1/2} f(t)e^{\sin \pi t}dt=\int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt$

61. If f(x) is continuous and differentiable function and $f\left(\frac{1}{n}\right)=0 \forall n\geq 1$ and n ∈ I, then

(a) f(x) = 0, x ∈ (0, 1] (b) f(0) = 0, f ′(0) = 0
(c) f ′(0) = 0 = f ″(0), x ∈ (0, 1] (d) f(0) = 0 and f ′(0) need not be zero

62. For x > 0, $\lim_{x\to 0}\left((\sin x)^{1/x}+\left(1/x\right)^{\sin x}\right)$ is

(a) 0 (b) -1 (c) 1 (d) 2

ANSWERS(Limit, Continuity and Differentiability)

1.	c	**2.**	a	**3.**	b	**4.**	b	**5.**	d	**6.**	a
7.	d	**8.**	c	**9.**	c	**10.**	c	**11.**	b	**12.**	c
13.	c	**14.**	b	**15.**	a	**16.**	a	**17.**	c	**18.**	c
19.	b	**20.**	a	**21.**	b	**22.**	a	**23.**	a	**24.**	d

25.	a	**26.**	b	**27.**	a	**28.**	d	**29.**	b	**30.**	d
31.	b	**32.**	c	**33.**	a	**34.**	c	**35.**	b	**36**	a
37.	c	**38.**	d	**39.**	b	**40.**	d	**41.**	c	**42.**	**b**
43.	c	**44.**	a	**45.**	b	**46.**	a	**47.**	d	**48.**	**a**
49.	d	**50.**	c	**51**.	c	**52.**	a	**53**.	c	**54**.	d
55.	d	**56.**	a	**57.**	c	**58.**	a	**59.**	c	**60.**	a,b,c,d
61.	b	**62.**	c								

DIFFERENTIATION

1. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+........\infty}}}$, then the value of dy/dx =

(a) $\sqrt{\frac{\sin x}{y+1}}$ (b) $\frac{\sin x}{y+1}$ (c) $\frac{\cos x}{2y+1}$ (d) $\frac{\cos x}{2y-1}$

2. The derivative of f(x) = $|x|$ at x = 0 is

(a) 1 (b) 0 (c) -1 (d) it does not exist

3. If $x^y \cdot y^x = 1$, then dy/dx =

(a) $\frac{y(x\log y - y)}{x(y\log x - y)}$ (b) $\frac{y(x\log y - y)}{x(y\log x + x)}$

(c) $\frac{y(x\log y + y)}{x(y\log x - x)}$ (d) $-\frac{y(x\log y + y)}{x(y\log x + x)}$

4. If $\sqrt{(x^2+y^2)} = ae^{\tan^{-1}(y/x)}, a>0$. Then y″(0), equals

(a) $\frac{a}{2}\cdot e^{\pi/2}$ (b) $ae^{\pi/2}$ (c) $-\frac{2}{a}\cdot e^{-\pi/2}$ (d) $\frac{a}{2}\cdot e^{-\pi/2}$

5. If $ax^2 + 2hxy + by^2 = 1$, then $\frac{d^2y}{dx^2}$ equals

(a) $\frac{h^2+ab}{(hx+by)^2}$ (b) $\frac{h^2-ab}{(hx+by)^2}$

(c) $\frac{h^2+ab}{(hx+by)^3}$ (d) $\frac{h^2-ab}{(hx+by)^3}$

6. The equation of $\frac{dy}{dx}$ the function $y = a^{x^{a^{x^{\cdots\infty}}}}$ is

(a) $\frac{y^2}{x(1-y\log x)}$ (b) $\frac{y^2\log y}{x(1-y\log x)}$

(c) $\frac{y^2\log y}{x(1-y\log x\log y)}$ (d) $\frac{y^2\log y}{x(1+y\log x\log y)}$

7. If $y^2 = ax^2 + bx + c$, then $y^3\cdot\frac{d^2y}{dx^2}$ is

(a) a constant (b) a function of x only
(c) a function of y only (d) a function of x and y

8. If x = a(cos θ + θ sin θ), y = a(sin θ - θ cos θ, then $\frac{dy}{dx}$ =

(a) cos θ (b) tan θ (c) sec θ (d) cosec θ

9. If f(x) = $\log_x$ (log x), then f′(x) at x = e is

(a) 0 (b) 1 (c) 1/e (d) ½ e

10. The differentiable coefficient of f(log x) w.r.t x, where f(x) = log x is

(a) x/log x (b) (log x)/x (c) (x log x)-1 (d) none of these

11. If $y = x^{x^{x^{x^{\infty}}}}$, then x(1 – y log x) $\frac{dy}{dx}$ is

(a) x^2 (b) y^2 (c) xy^2 (d) none of these

12. If f(x) = x + 2, then f′(f(x)) at x = 4 is

(a) 8 (b) 1 (c) 4 (d) 5

13. If $f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}, 0 \le x \le \frac{\pi}{2}$, then $f'\left(\frac{\pi}{6}\right)$ is

(a) -1/4 (b) -1/2 (c) ¼ (d) ½

14. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2^x+2^y}{2^x-2^y}$ (b) $\frac{2^x+2^y}{1+2^{x+2}}$ (c) $\left(2^{x-y}\right)\cdot\frac{2^y-1}{1-2^x}$ (d) $\frac{2^{x+y}-2^x}{2^y}$

15. If $y=\cos^{-1}\left(\frac{1-\ln x}{1+\ln x}\right)$, then $\frac{dy}{dx}$ at x = e is

(a) - 1/e (b) -1/2e (c) 1/2e (d) 1/e

16. If y is a function of x and log(x + y) = 2xy, then the value of y′(0) is equal to:

(a) 1 (b) -1 (c) 2 (d) 0

17. If f(x) = x^n then the value of $f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}+\frac{f'''(1)}{3!}+.......\frac{(-1)^n f^n(1)}{n!}$

is

(a) 1 (b) 2n (c) 2n – 1 (d) 0

18. The solution of the differential equation $(1 + y^2) + \left(x-e^{\tan^{-1}x}\right)\frac{dy}{dx}=0$

(a) $xe^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$ (b) $(x-2)=ke^{\tan^{-1}y}$

(c) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$ (d) $xe^{\tan^{-1}y}=\tan^{-1}y+k$

19. If $x=e^{y+e^{y+.....to\infty}}$, x > 0, then $\frac{dy}{dx}$ is

(a) $\frac{1+x}{x}$ (b) 1/x (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

20. If $x^m\cdot y^n=(x+y)^{m+n}$, then dy/dx is

(a) xy (b) x/y (c) y/x (d) $\frac{x+y}{xy}$

21. If $xe^{xy} = y + \sin^2 x$, then at x = 0, dy/dx =

(a) 1 (b) 2 (c) 4 (d) 3

22. If $x^2 + y^2 = 1$, then

(a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$

(c) $yy'' - (y')^2 – 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$

23. If y is function of x and log(x + y) – 2xy = 0, then the value of y′(0) is equal to

(a) 1 (b) -1 (c) 2 (d) 0

24. If $f''(x)=-f(x), g(x)=f'(x), F(x)=\left(f\left(\frac{x}{2}\right)\right)^2+\left(g\left(\frac{x}{2}\right)\right)^2$ and F(5) = 5,

then

F(10) is equal to

(a) 5 (b) 10 (c) 0 (d) 15

25. $\frac{d^2x}{dy^2}$ equals

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ **(b)** $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ **(d)** $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

ANSWERS(Differentiation)

1.	d	**2.**	d	**3.**	d	**4.**	c	**5.**	d	**6.**	c
7.	a	**8.**	b	**9.**	b	**10.**	c	**11.**	c	**12.**	b
13.	d	**14.**	c	**16.**	a	**17.**	d	**18.**	c	**19.**	c
20.	c	**21**.	a	**22**.	b	**23.**	a	**24.**	a	**25**.	b

Application of Derivative – I

1. The value of k in order that f(x) = sin x – cos x – kx + b decreases for all real values, is given by (a) $k < 1$ (b) $k > 1$ (c) $k > 1$ (d) $k < 2$

2. f(x) = x(x – 2) (x – 4), will satisfy mean value theorem at
(a) 1 (b) 2 (c) 3 (d) 4

3. The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$ is strictly increasing for all x, if

(a) $k \geq 3/2$ (b) $k > 3/2$ (c) $k < 3/2$ (d) $k \leq 3/2$

4. The set of all values of a for which the function $f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$ decreases for all real x, is

(a) $(-\infty, \infty)$ (b) $(1, \infty)$

(c) $\left(-3, \frac{5-\sqrt{27}}{2}\right) \cup (2, \infty)$ (d) $\left(-4, \frac{3-\sqrt{21}}{2}\right) \cup (1, \infty)$

5. For the given integer k, in the interval $\left[2\pi k - \frac{\pi}{2}, 2\pi k + \frac{\pi}{2}\right]$ the graph of sin x is

(a) increasing from -1 to 1 (b) decreasing from -1 to 0

(c) decreasing from 0 to 1 (d) none of these

6. The length of the sub tangent to the curve $x^2 + xy + y^2 = 7$ at (1, -3) is

(a) 3 (b) 5 (c) 15 (d) 3/5

7. If $f(x) = 2x^6 + 3x^4 + 4x^2$ then $f'(x)$ is

(a) even function (b) an odd function

(c) neither even nor odd (d) none of these

8. Normal at a point to the parabola $y^2 = 4ax$, when abscissa is equal to ordinate, will meet the parabola again at a point

(a) (6a, -9a) (b) (-9a, 6a) (c) (-6a, 9a) (d) (9a, -6a)

9. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is:

(a) π/6 (b) π/4 (c) π/3 (d) π/2

10. For what value of *a*, $f(x) = -x^2 + 4ax^2 + 2x - 5$ is decreasing ∀x.

(a) (1, 2) (b) (3, 4) (c) R (d) no value of *a*

11. The common tangent of the parabolas $y^2 = 4x$ and $x^2 = -8y$ is

(a) y = x + 2 (b) y = x – 2 (c) y = 2x + 3 (d) none of these

12. The function $f(x) = x(x+3)e^{-(1/2)x}$ satisfies all the conditions of Roll's theorem in $[-3, 0]$.

The value of c is

(a) 0 (b) -1 (c) -2 (d) -3

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13. $f(x)=\left(\frac{e^{2x}-1}{e^{2x}+1}\right)$ is

(a) an increasing (b) a decreasing (c) an even (d) None of these

14. A point on the parabola $y^2 = 18x$ at which ordinate increases at twice the rate of abscissa is

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$ (c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) (2, 4)

15. A function y = f(x) has a second order derive $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is y = 3x – 5, then the function is

(a) $(x + 1)^2$ (b) $(x - 1)^2$ (c) $(x + 1)^3$ (d) $(x - 1)^3$

16. The normal of the curve x = a(1 + cos θ), y = a sin θ at 'θ' always passes through the fixed point

(a) (a, a) (b) (0, a) (c) (0, 0) (d) (a, 0)

17. The normal to the curve x = a (cos θ + θ sin θ), y = a(sin θ - θ cos θ) at any point 'θ' is such that

(a) it passes through the origin (b) it makes angle π/2 + θ with the x-axis

(c) it passes through $\left(a\frac{\pi}{2}, \theta-\alpha\right)$ (d) it is constant distance from the origin

18. a spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(a) $\frac{1}{36\pi} cm/\min.$ (b) $\frac{1}{18\pi} cm/\min$

(c) $\frac{1}{54\pi} cm/\min$ (d) $\frac{5}{6\pi} cm/\min$

19. If the equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0,\ a_1 \neq 0, n \geq 2$, has a positive root x = α, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... a_1 = 0$ has a positive root, which is

(a) greater than α (b) smaller than α

(c) greater than or equal to α (d) equal to α

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20. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at points (2, 0) and (3, 0) is

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

21. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $(-\pi/2, \pi/4)$ (b) $(0, \pi/2)$ (c) $(-\pi/2, \pi/2)$ (d) $(\pi/4, \pi/2)$

22. Consider the following statements S and R

S: Both sin x and cos x are decreasing functions in the interval $\left(\frac{\pi}{2}, \pi\right)$

R: If a differentiable function decreases in the interval (a, b), then its derivative also decreases in (a, b). Which of the following is true?

(a) Both S and R are wrong

(b) Both S and R are correct, but R is not the correct explanation of S

(c) S is correct and R is the correct explanation for S

(d) S is correct and R is wrong

23. If $f(x) = xe^{x(1-x)}$, then f(x) is

(a) increasing on [- ½ , 1] (b) decreasing on IR

(c) increasing on IR (d) decreasing on [- ½ , 1]

24. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes, lies in the first quadrant. If its area is 2, then the value of b is

(a) -1 (b) 3 (c) -3 (d) 1

25. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is (are)

(a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (b) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ (c) (0, 0) (d)* $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

26. The length of a longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is

(a) $\pi/3$ (b) $\pi/2$ (c) $3\pi/2$ (d) π

27. In [0, 1] Lagranges Mean Value theorem is NOT applicable to

(a) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$ (b) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(c) $f(x) = x|x|$ (d) $f(x) = |x|$

28. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

(a) f(x) is a strictly increasing function (b) f(x) has a local maxima

(c) f(x) is a strictly decreasing function (d) f(x) is bounded

29. If $f(x)= x^{\alpha} \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in
[0, 1] is
(a) -2 (b) -1 (c) 0 (d) ½

30. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$

(a) on the left of x = c **(b)** on the right of x = c
(c) at no point **(d)** at all points

ANSWERS(Application of Derivative-I)

14.	c	**15.**	c	**16.**	b	**17.**	d	**18.**	a	**19.**	c
20.	b	**21.**	d	**22.**	d	**23.**	d	**24.**	d	**25.**	c
13	a	**14.**	a	**15.**	b	**16.**	d	**17.**	d	**18.**	b
19.	b	**20.**	d	**21.**	a	22.	d	23.	a	24.	c
25.	d	**26.**	a	**27.**	a	**28.**	a	**29.**	d	**30.**	a

Application of Derivatives – II (Maxima and Minima)

1. The least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is

(a) $4\sqrt{3r}$ (b) $2\sqrt{3r}$ (c) $6\sqrt{3}\ r$ (d) $8\sqrt{3r}$

2. The minimum value of $|x-3|+|x-2|+|x-5|$ is

(a) 3 (b) 7 (c) 15 (d) 0

3. For real x, maximum value of $\frac{x^2-x+1}{x^2+x+1}$ is

(a) ½ (b) 2 (c) 1/3 (d) 3

4. If the function f(x) = $x^3 + \alpha^2 x^2 + \beta x + 1$ has maximum value at x = 0 and minimum at x = 1, then

(a) $\alpha = 2/3, \beta = 0$ (b) $\alpha = -3/2, \beta = 0$ (c) $\alpha = 0, \beta = 3/2$ (d) none of these

5. f(x) = $f(x)=\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$. The point of extrema of the function in the interval (1, 3) is

(a) x = 1 (b) x = 2 (c) x = 2.1 (d) x = 1.5

6. The greatest value of f(x) = 2 sin x + sin 2x, on [0, 3π/2], is given by

(a) 9/2 (b) 5/2 (c) $\frac{3\sqrt{3}}{2}$ (d) 3/2

7. Let f(x) = cos x sin 2x. Then

(a) min {f(x); (-π ≤ x ≤ π) > - 8/9} (b) min {f(x); (-π ≤ x ≤ π) > - 3/7}

(c) min {f(x); (-π ≤ x ≤ π) > - 1/9} (d) min {f(x); (-π ≤ x ≤ π) > - 2/9}

8. The maximum value of 12 sin θ - 9 $\sin^2$ θ is

(a) 3 (b) 4 (c) 5 (d) none of these.

9. What are the minimum and maximum values of the function f(x) = $x^5 - 5x^4 + 5x^3 - 10$?

(a) -37, -9 (b) -10, 0 (c) it has 2 min. and 1 max. values

(d) it has 2 max. and 1 min. values

10. What is the maximum value of $(1/x)^x$?

(a) $(e)^{1/e}$ (b) $(1/e)^e$ (c) e^{-e} (d) none of these

11. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0 (b) ¼ (c) ½ (d) 1/3

12. $|\sin x + \cos x|$

(a) ≤ 2 (b) ≥ 2 (c) $\le \sqrt{2}$ (d) $\le 1/\sqrt{2}$

13. Minimum value of $(3 \sin \theta - 4 \cos \theta + 7)^{-1}$ is

(a) 1/12 (b) 5/12 (c) 7/12 (d) 1/6

14. The minimum value of $f(a) = (2a^2 - 3) + 2(3 - a) + 4$

(a) 15/2 (b) 11/2 (c) -13/2 (d) 13/2

15. The number of values of x where $f(x) = \cos x + \cos \sqrt{2}x$ attains its maximum value is

(a) 1 (b) 0 (c) 2 (d) infinite.

16. If $y = a \ln x + bx^2 + x$ has its extremum values at x = -1 and x = 2, then :

(a) a = 2, b = -1 (b) a = 2, b = -1/2 (c) a = -2, b = ½ (d) none

17. The maximum value of $\cos^2\left(\frac{\pi}{3} - x\right) - \cos^2\left(\frac{\pi}{3} + x\right)$ is

(a) $-\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{2}$

18. If $f(x) = 2x^3 - 21x^2 + 36x - 30$, then which one of the following is correct ?

(a) $f(x)$ has minimum at x = 1 (b) $f(x)$ has maximum at x = 6

(c) $f(x)$ has maximum at x = 1 (d) $f(x)$ has no maxima or minima

19. The maximum distance from origin of a point on the curve $x = a \sin t - b \sin\left(\frac{at}{b}\right)$, $y = a \cos t - b \cos\left(\frac{at}{b}\right)$, both a, b > 0 is

(a) a – b (b) a + b (c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

20. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(a) ½ (b) 3 (c) 1 (d) 2

21. The positive number x when added to its inverse gives the minimum value of the sum at x equal to

(a) -2 (b) 2 (c) 1 (d) -1

22. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) 2ab (b) ab (c) $\sqrt{ab}$ (d) a/b

23. The function $f(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at

(a) x = 0 (b) x = 1 (c) x = 2 (d) x = -2

24. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(a) 1 (b) 17/7 (c) ¼ (d) 41

25. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(a) ½ x^2 (b) πx^2 (c) 3/2 x^2 (d) $\sqrt{\frac{x^3}{8}}$

26. If f(x) = $x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then (-∞, ∞)

(a) f(x) is a strictly increasing function (b) f(x) has a local maxima
(c) f(x) is a strictly decreasing function (d) f(x) is bounded

27. The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and coordinate axes is

(a) ab (b) $\frac{a^2+b^2}{2}$

(c) $\frac{(a+b)^2}{2}$ (d) $\frac{a^2+ab+b^2}{3}$

28.** f(x) is cubic polynomial which has local maximum at x = -1. If (2) = 18, f(1) = -1, f(x) has local minima at x = 0, then

(a) the distance between (-1, 2) and (a, f(a)) where x = -a is the point of local minima is $2\sqrt{5}$

(b) f(x) is increasing for x ∈ [1, $2\sqrt{5}$] (c) f(x) has local minima at x = 1

(d) the value of f(0) = 5

29.** $f(x)=\begin{cases} e^x & , 0\le x\le 1 \\ 2-e^{x-1}, & 1<x\le 2 \\ x-e & , 2<x\le 3 \end{cases}$ and $g(x)=\int_0^x f(t)\,dt, x\in[1,3]$ then

(a) g(x) has local maxima at x = 1 + ln 2 and local minima at x = e
(b) f(x) has local maxima at x = 1 and local minima at x = 2
(c) f(x) has no local maxima (d) no local minima

30. If f(x) is a twice differentiable function such that f(a) = 0, f(b) = 2, f(c) = -1, f(d) = 2, f(e) = 0. Where a < b < c < d < e. then the minimum number of zeros of g(x) = $(f'(x))^2 + f''(x)f(x)$ in the interval [a, e] is

(a) 5 (b) 6 (c) 7 (d) 8

ANSWERS(Application of Derivative-II)

1.	c	**2.**	a	**3.**	d	**4.**	b	**5.**	b	**6.**	c
7.	a	**8.**	b	**9.**	a	**10.**	a	**11.**	b	**12.**	c
13.	a	**14.**	d	**15.**	a	**16.**	b	**17.**	c	**18.**	c
19.	b	**20.**	d	**21.**	c	**22.**	a	**23.**	c	**24.**	d
25.	a	26.	a	**27.**	a	**28**.	b,d	**29.**	a,b	**30.**	b

MATHEMATICS

Integral Calculus

SECTION - A

1. Integrate the following functions

(i) $\cos^2 2x \sin 3x \sin 5x$ (ii) $\cos 2x\left(\dfrac{\cos x + \sin x}{\cos x - \sin x}\right)$

(iii) $\sin^6 x$ (iv) $\dfrac{1}{1+\tan x}$

(v) $\dfrac{e^x - 1}{e^x + 1}$

2. (i) $\dfrac{1}{1+\sin^2 x}$ (ii) $\dfrac{\sin 2x}{\sin^4 x + \cos^4 x}$

(iii) $\dfrac{1}{\sin x + \cot x}$ (iv) $\sqrt{1+\sec x}$

(v) $\dfrac{1}{\sin^2 x + \tan^2 x}$

3. (i) $\cot^3 x \operatorname{cosec}^4 x$ (ii) $(e^x - 1)^{1/2}$

(iii) $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)^{1/2}$ (iv) $\sqrt{\tan x}$

(v) $\sec^3 x\sqrt{\operatorname{cosec} 2x}$

4. (i) $\dfrac{1}{2x^2 - 3x + 5}$ (ii) $\dfrac{1}{3x^2 - 2x - 1}$

(iii) $\dfrac{1}{\sqrt{2x^2 - x + 1}}$ (iv) $\dfrac{1}{\sqrt{2 - 3x - 5x^2}}$

(v) $\dfrac{2x+1}{3x^2 - 2x + 1}$

5. (i) $\dfrac{5x - 11}{\sqrt{2x^2 - 3x + 3}}$ (ii) $\dfrac{x^2}{x^2 + 7x + 10}$

(iii) $\dfrac{x^2 + 2}{x^3 - 1}$ (iv) $\dfrac{1}{(x+1)^2(x+2)}$

(v) $\frac{1}{x^4 - x^2}$

6. (i) $\frac{x^2}{(x+1)^2(x+2)}$ (ii) $\frac{x^2+2}{(x^2+3)(x^2+4)}$

(iii) $\frac{x}{(1+x)(1+x^2)}$ (iv) $\frac{2x+1}{3x^2-2x+1}$

(v) $\frac{1}{(x+1)^2(x+2)}$

7. (i) $x^2 \sin^3 x$ (ii) $e^{5x\cos^3 5x}$

(iii) $e^x \frac{x^2}{(x+2)^2}$ (iv) $e^x \frac{1+\sin x}{1+\cos x}$

(v) $e^{-x/2}\left(\frac{1-\sin x}{1+\cos x}\right)^{1/2}$

8. (i) $\frac{1}{\sin x + \sin 2x}$ (ii) $\frac{2-\sin x}{2+\sin x}$

(iii) $\frac{3\sin x - 4\cos x}{2\sin x + \cos x}$ (iv) $\left(\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right)^{1/2}$

(v) $\frac{\tan x}{1+\tan x+\tan^2 x}$

9. (i) $\int \frac{1}{1-\sin^4 x}dx$ (ii) $\int \frac{1}{(2\sin x + 3\cos x)}dx$

(iii) If $0 < a < 1$ then find $\int \frac{dx}{1-2a\cos x + a^2}$

10. (i) $\int e^{a^x} \sin bx\, dx$ (ii) $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

(iii) $\int x \log\left(1+\frac{1}{x}\right)dx$ (iv) $\int \frac{x^3-6x^2+10x-2}{x^2-5x+6}dx$

(v) $\int \frac{x^4+4x^3+11x^2+12x+8}{(x^2+2x+3)^2(x+1)}dx$

11. (i) $\int_0^1 x^3(1+3-x^4)^{1/4}\,dx$ (ii) $\int_0^{2a} (2ax-x^2)^{1/2}\,dx$

(iii) $\int_0^1 x^2 \tan^{-1} x\,dx$ (iv) $\int_0^{\pi} \frac{\sin 4x}{\sin x}dx$

(v) $\int_0^{\pi/2} \frac{\sin x + \cos x}{\sin 4x + \cos 4x}dx$

12. (i) $\int_0^{\pi/4} \frac{dx}{2 + \tan x}$ (ii) $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin x 2x}dx$

(iii) $\int_0^{a} \frac{dx}{x + (a^2 - x^2)^{1/2}}$ (iv) $\int_0^{\pi/4} \frac{x \sin x}{\cos^3 x}dx$

13. (i) $\int_0^{2} | x^2 + 2x - 3 | dx$ (ii) $\int_0^{1.5} (x^2) dx$

(iii) $\int_0^{a} [x^4] dx$ (iv) $\int_{-1}^{1} | x | dx$

14. (i) $\int_0^{\pi/2} \log \sin x\,dx$ (ii) $\int_0^{1} \cot^{-1}(1 - x + x^2)\,dx$

(iii) $\int_0^{\pi/2} \frac{dx}{1 + \frac{1}{6}\sin^2 x}$ (iv) $\int_0^{1} (1 - x^2)^{3/2}\,dx$

15. (i) $\int_0^{\infty} \frac{x \log x}{(1 + x^2)^2}dx$ (ii) $\int_0^{\infty} \frac{x^2}{1 + x^4}dx$

(iii) $\int_0^{\pi} \frac{\sqrt{1 - x^2}}{1 - x^2 \sin^2 \alpha}dx$ (iv) $\int_0^{\pi} \log(1 + \cos x)dx$

16. (i) $\int_0^{\pi/2} \left| \frac{1}{2} - \sin x \right| dx$ (ii) $\int_0^{\pi} | \cos x | dx$

(iii) $\int_{\pi/6}^{\pi/3} | \tan x - \cot x | dx$ (iv) $\int_{-\pi/2}^{\pi/2} (\cos x - \cos^3 x)\,dx$

17. (i) $\int_0^{\pi} \left| \frac{1}{2} - \cos x \right| dx$ (ii) $\int_0^{\pi/2} \left| \frac{1}{4} - \sin^{-1} x \right| dx$

(iii) $\int_0^{\pi} || \sin x | - | \cos x || dx$ (iv) $\int_0^{\pi/2} | \sin x - \cos x | dx$

18. (i) $\int_0^3 [x]dx$ (ii) $\int_0^{1.5} [x^2]dx$

(iii) $\int_{-1}^1 x-[x]dx$ (iv) $\int_0^2 [x^2]dx$

19. (i) $\int_0^2 [x-x^2]dx$ (ii) $\int_0^2 [x^2-x+1]dx$

(iii) $\int_0^{3\pi} [\sin x]dx$ (iv) $\int_0^{5\pi/2} [\tan x]dx$

20. If $f(x)=\int_1^x \frac{t^4 \sin\frac{1}{t}+t^2}{1+|t|^3}dt$, then prove that $\lim_{x\to-\infty} f'(x)=-1$

21. Prove that $\int \frac{1}{a^2\sin^2 x+b^2\cos^2 x}dx = \frac{\pi(a^2+b^2)}{4a^3b^3}$

22. Prove that $\int_{e^{-1}}^{\tan x} \frac{1}{1+t^2}dt+\int_{e^{-1}}^{\cot x} \frac{dt}{1+t^2}=1$

23. $f(x)=\int_1^x \frac{\log t}{t+1}dt \ \ x>0$. Prove that $f(x)+f\left(\frac{1}{x}\right)=\frac{1}{2}(\log x)^2$

24. If $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx = fog(x)+\cos x$

25. If $I_{m,n}=\int \cos^m x\cos nx\,dx$ show that $(m+n)I_{m,n}=\cos^m x\sin nx+m\,I_{m-1,n-1}$

26. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax+B\log(9e^{2x}-4)+c$, then find A, B and C

27. If $f:R\to R$ is differentiable and f(1) = 4 then prove that $\lim_{x\to 1}\int_4^{f(x)} -2t\,dt=8f'(1)$

28. If $g(x)=\int_0^x \cos^4 x\,dx$ then show that $g(x+\pi)=g(x)+g(\pi)$

29. Evaluate $\int_{-1}^1 (x-[x])dx$

30. Evaluate $\int_0^{\pi/4} \log(1+\tan x)dx$

31. Evaluate $\int_{-1/\sqrt{3}}^{2/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$

32. (i) $\int_2^3 \frac{\sqrt{x}\,dx}{\sqrt{5-x}+\sqrt{x}}$ (ii) $\int_{\pi/4}^{3\pi/4} \frac{\phi\, d\phi}{1+\sin\phi}$ (iii) $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$

Answers

16. (i) $\sqrt{3}-1\frac{-\pi}{12}$ (ii) 2π (iii) $\log\frac{4}{3}$ (iv) $\frac{4}{3}$

17. (i) $\sqrt{3}+\frac{\pi}{6}$ (ii) $\frac{1}{2}\sin^{-1}\frac{1}{4}-1$ (iii) $4(\sqrt{2}-1)$ (iv) $2\sqrt{2}-2$

18. (i) 3 (ii) $2\sqrt{3}$ (iii) -1 (iv) $5-\sqrt{2}-\sqrt{3}$

19. (i) $\frac{\sqrt{5}-5}{2}$ (ii) $\frac{5-\sqrt{5}}{2}$ (iii) 0 (iv) $\frac{\pi}{4}$

24. $f(x)=\frac{x^2}{2}g(x)=\log(x+\sqrt{1+x^2})$

SECTION – B

1. $\int \frac{dx}{1+e^x} =$

(a) $\log(1+e^x)+c$ (b) $-\log(1+e^{-x})+c$

(c) $-\log(1-e^x)+c$ (d) $\log(e^x+e^{-2x})+c$

2. $\int \sqrt{1-\sin 2x}\,dx =$, where $x \in \left(0, \frac{\pi}{4}\right)$

(a) $-\sin x+\cos x+c$ (b) $\sin x-\cos x+c$

(c) $\tan x+\sec x+c$ (d) $\sin x+\cos x+c$

3. $\int \frac{1+\cos^2 x}{\sin^2 x}dx =$

(a) $\cot x-2x+c$ (b) $-2\cot x+2x+c$

(c) $-2\cot x-x+c$ (d) $-2\cot x+x+c$

4. $\int \frac{1}{e^x+e^{-x}}dx =$

(a) $\tan^{-1}(e^x)+c$ (b) $\tan^{-1}(e^{-x})+c$

(c) $\log(e^x+e^{-x})+c$ (d) $\log(e^x-e^{-x})+c$

5. $\int \frac{dx}{x(1+\log x)} =$

(a) $\log(1+\log x)$
(b) log(log(1 + logx))
(c) logx + log(logx)
(d) none of these

6. $\int \frac{x\,dx}{\sqrt{4-x^4}} =$

(a) $-\cos^{-1}\left(\frac{x^2}{2}\right)$
(b) $\frac{1}{2}\cos^{-1}\left(\frac{x^2}{2}\right)$
(c) $-\sin^{-1}\left(\frac{x^2}{2}\right)$
(d) $\frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right)$

7. To find the value of $\int \frac{1+\log x}{x} dx$, a proper substitution is

(a) logx = t
(b) 1 + logx = t
(c) $\frac{1}{x} = t$
(d) none of these

8. $\int x \sec^2 x\,dx =$

(a) $\tan x^2$
(b) $\tan^2 x$
(c) x tanx – log snx
(d) x tanx + log cosx

9. $\int x \sin x\,dx = -x\cos x + A$, then A =

(a) sinx + c
(b) cosx + c
(c) c
(d) none of these

10. $\int \frac{\sin 2x}{1+\sin^2 x} dx =$

(a) $\log \sin 2x + c$
(b) $\log(1+\sin^2 x) + c$
(c) $\frac{1}{2}\log(1+\sin^2 x) + c$
(d) $\tan^{-1}(\sin x) + c$

11. $\int_0^{\pi/4} \frac{\sec x\,dx}{1+2\sin^2 x} =$

(a) $\frac{1}{3}\left[\log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}}\right]$
(b) $\frac{1}{3}\left[\log(\sqrt{2}+1) - \frac{\pi}{2\sqrt{2}}\right]$
(c) $3\left[\log(\sqrt{2}+1) - \frac{\pi}{2\sqrt{2}}\right]$
(d) $3\left[\log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}}\right]$

12. $\int_0^{\pi} e^{\cos^2 x} \cos^5 3x \, dx =$

(a) 1 (b) -1 (c) 0 (d) none of these

13. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx =$

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) 1

14. The area bounded by curve $y^2 = x$, line $y = 4$ and y-axis is

(a) $\frac{16}{3}$ (b) $\frac{64}{3}$ (c) $\sqrt{2}$ (d) none of these

15. $\int_1^2 \log x \, dx =$

(a) $\log\left(\frac{1}{e}\right)$ (b) log4 (c) $\log\frac{4}{e}$ (d) log2

16. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f\,|x|\,dx = \frac{2A}{\pi}$, the values of A and B are respectively

(a) $\frac{\pi}{2}$ and $\frac{\pi}{3}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ (c) $\frac{4}{\pi}$ and 0 (d) 0 and $\frac{4}{\pi}$

17. The value of $\int_{\pi}^{2\pi} [2\sin x] dx$, where [ ] is greatest integer function, is

(a) $-\pi$ (b) -2π (c) $-\frac{5\pi}{3}$ (d) $\frac{5\pi}{3}$

18. For evaluating the integral $\int_{-2}^{2} (px^2 + qx + s) dx$, the value the following constants are

(a) p (b) q (c) s (d) p and s

19. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x) dx =$

(a) 2/15 (b) 4/15 (c) 6/15 (d) 8/15

20. The area bounded by the curve y = sinx, y = 0, x = 0 and $x = \pi/2$ is

(a) π (b) 2π (c) 1 (d) 2

SECTION – C

1. Let $f:R\to R$ and $g:R\to R$ be continuous function. Then the value of the integral

$$\int_{-\pi/2}^{\pi/2}[f(x)+f(-x)][g(x)-g(-x)]dx \text{ is}$$

(a) π (b) 1 (c) -1 (d) 0

2. $$\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\,dt}{x\sin x}=$$

(a) 1 (b) 2 (c) 4 (d) none of these

3. The value of the definite integral $\int_0^1(1+e^{-x^2})dx$ is

(a) -1 (b) 2 (c) $1+e^{-1}$ (d) none of these

4. Solution of the equation $\int_3^x\sqrt{x+1}\,dx=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

5. The value of $\int_{-2}^{2}\frac{dx}{1+|x-1|}$

(a) log2 (b) 2 log2 (c) 3 log2 (d) 4 log2

6. $\int_0^{\pi/4}\frac{(\sin\theta+\cos\theta)}{(9+16\sin 2\theta)}$ equals

(a) $\frac{1}{10\log 2}$ (b) $\frac{1}{20\log 5}$ (c) $\frac{1}{20}\log 3$ (d) $\frac{1}{30\log 7}$

7. $\int_1^x\frac{\log(x^2)}{x}dx$

(a) $(\log x)^2$ (b) $\frac{1}{2}(\log x)^2$ (c) $\frac{\log x^2}{2}$ (d) none of these

8. The value of integral $\int_0^{\pi} x f(\sin x) dx$ is

(a) 0 (b) $\pi\int_0^{\pi} f(\sin x) dx$ (c) $\frac{\pi}{2}\int_0^{\pi} f(\sin x) dx$ (d) none of these

9. $\int_1^{x} \frac{\log(x^2)}{x} dx$

(a) $(\log x)^2$ (b) $\frac{1}{2}(\log x)^2$ (c) $\frac{\log x^2}{2}$ (d) none of these

10. Let $f : R \to R$ be differentiable function and f(1) = 4. Then the value of $\lim_{x \to t} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

(a) $8f'(1)$ (b) $4f'(1)$ (c) $2f'(1)$ (d) $f'(1)$

11. The area enclosed within the curve |x| + |y| = 1 is

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 4

12. The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) 6π (b) $\frac{\pi(a^2 + b^2)}{4}$ (c) $\pi(a + b)$ (d) $\frac{\pi ab}{4}$

13. Area of the region bounded by the curve y = tanx, tangent drawn to the curve at $x = \pi/4$ and the x-axis is

(a) $\log\sqrt{2}$ (b) $\log\sqrt{2} + \frac{1}{4}$ (c) $\log\sqrt{2} - \frac{1}{4}$ (d) $\frac{1}{4}$

14. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}$

(a) π (b) $a\pi$ (c) $\pi/2$ (d) 2π

15. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is

(a) 3/2 (b) 5/2 (c) 3 (d) 5

16. The area bounded by the curves y = f(x), the x-axis and the ordinates x = 1 and x = b is $(b-1)\sin(3b+4)$. Then f(x) is

(a) $(x-1)\cos(3x+4)$ (b) $\sin(3x+4)$

(c) $\sin(3x+4)+3(x-1)\cos(3x+4)$ (d) none of these

17. Let $f:(0,\infty)\to R$ and $F(x)=\int_1^x f(t)\,dt$. If $F(x^2) = x^2(1 + x)$, then f(4) equals

(a) 5/4 (b) 7 (c) 4 (d) 2

18. Let $g(x)=\int_0^x f(t)\,dt$ where f is such that $\frac{1}{2}\le f(t)\le 1$ for $t\in[0, 1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1, 2]$. Then g(2) satisfies the inequality

(a) $-\frac{3}{2}\le g(2)\le\frac{1}{2}$ (b) $0\le g(2)<2$

(c) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (d) $2 < g(2) < 4$

19. If $g(x)=\int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(a) $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ (c) $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

20. If for a real number y, [y] is the greatest integer $\le y$, then $\int_{\pi/2}^{3\pi/2}[2\sin x]\,dx$ is

(a) $-\pi$ (b) 0 (c) $-\pi/2$ (d) $\pi/2$

21. If $f(x)=\begin{cases} e^{\cos x}\sin x & \text{for } |x|\le 2 \\ 2 & \text{otherwise}\end{cases}$, then $\int_{-2}^{3} f(x)\,dx =$

(a) 0 (b) 1 (c) 2 (d) 3

22. $\int e^{\tan^{-1}}\left(\frac{1+x+x^2}{1+x^2}\right)dx$

(a) $\frac{xe^{\tan^{-1}x}}{(1+x^2)^2}$ (b) $xe^{\tan^{-1}x}$ (c) $\frac{xe^{\tan^{-1}x}}{1+x^2}$ (d) none of these

23. The value of the integral $\int_0^{\alpha}\frac{x\,dx}{1+\cos\alpha\sin x}$, $0<\alpha<\pi$ is

(a) $\frac{\pi\alpha}{\sin\alpha}$ (b) $\frac{\pi\alpha}{1+\sin\alpha}$ (c) $\frac{\pi\alpha}{\cos\alpha}$ (d) $\frac{\pi\alpha}{1+\cos\alpha}$

24. If $f(x) = ae^{2x} + be^{x} + cx$, satisfies the conditions f(0) = -1

$f'(\log 2) = 31$, $\int_0^{\log 4} [f(x) - x]dx = \frac{39}{2}$, then

(a) a = 5, b = 6, c = 6 (b) a = 5, b = −6, c = 3

(c) a = −5, b = 6, c = 3 (d) none of these

25. $\int_0^1 \frac{\sin x + x^2}{3 - |x|}dx$ is equal to

(a) $\int_0^1 \frac{\sin x + x^2}{3 - |x|}dx$ (b) 0 (c) $\int_0^1 \frac{x^2}{3 - |x|}dx$ (d) $\int_0^1 \frac{\sin x}{3 - |x|}dx$

26. Let f be a positive function, let $I_1 = \int_{1-k}^{k} x f(x(1-x))dx$, $I_2 = \int_{1-k}^{k} f(x(1-x))dx$, where 2k – 1 > 0. Then $\frac{I_1}{I_2}$ is

(a) 2 (b) k (c) 1/2 (d) 1

27. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}}dx =$

(a) $a^{\sqrt{x}} \log_e a + c$ (b) $2a^{\sqrt{x}} \log_e a + c$

(c) $2a^{\sqrt{x}} \log_{10} a + c$ (d) $2a^{\sqrt{x}} \log_a e + c$

28. The value of the definite integral $\int_0^1 (1 + e^{-x^2})dx$ is

(a) -1 (b) 2 (c) $1 + e^{-1}$ (d) none of these

29. $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx$ is equal to

(a) $\frac{5^{5^x}}{(\log 5)^3} + c$ (b) $5^{5^{5^x}} (\log 5)^3 + c$ (c) $\frac{5^{5^{5^x}}}{(\log 5)^3} + c$ (d) none of these

30. The indefinite integral $\int (\sqrt{\tan x} + \sqrt{\cot x})dx$ is

(a) $\frac{\tan x}{\sqrt{2}}\tan^{-1}\left(\frac{\cot x+1}{\sqrt{2\tan x}}\right)$ (b) $\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)$

(c) $\sqrt{2}\tan^{-1}\left(\frac{\tan x+1}{\sqrt{2\tan x}}\right)$ (d) $\frac{\tan x}{\sqrt{2}}\tan^{-1}\left(\frac{\cot x-1}{\sqrt{2\cot x}}\right)$

Complex numbers

PROBLEM SET ON COMPLEX NUMBERS

1. The locus of the center of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be

(a) an ellipse (b)* a hyperbola (c) a circle (d) none of these

2. If z and w are two non – zero complex numbers such that $|zw| = 1$ and arg (z) – arg (w) = $\pi/2$, then $\bar{z}w$ is equal to

(a)* -i (b) 1 (c) -1 (d) I

3. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle. Then

(a) $a^2 = 4b$ (b) $a^2 = b$ (c) $a^2 = 2b$ (d)* $a^2 = 3b$

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$ n is a positive integer then

(a) x = 2n + 1 (b)* x = 4n (c) x = 2n (d) x = 4n + 1

5. Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. Then arg z equals

(a) $5\pi/4$ (b) $\pi/2$ (c)* $3\pi/4$ (d) $\pi/4$

6. If $z = x - iy$ and $z^{1/3} = a + ib$, then $\frac{\left(\frac{x}{a} + \frac{y}{b}\right)}{\left(a^2 + b^2\right)}$ is equal to

(a)* -2 (b) -1 (c) 2 (d) 1

7. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) an ellipse (b) the imaginary axis (c)* a circle (d) the real axis

8. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x - 1)^3 + 8 = 0$ are

(a) $-1, -1 + 2\omega, -1 - 2\omega^2$ (b) -1, -1, -1

(c)* $-1, 1 - 2\omega, 1 - 2\omega^2$ (d) $-1, 1 + 2\omega, 1 + 2\omega^2$

9. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then

arg z_1 – arg z_2 is equal to
(a) π/2 (b) -π (c)* 0 (d) -π/2

10. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on
(a) an ellipse (b) a circle (c)* a straight line (d) a parabola

11. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is
(a) -1 (b)* -i (c) i (d) 1

12. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of
$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \ldots\ldots + \left(z^6+\frac{1}{z^6}\right)^2$ is
(a) 6 (b)* 12 (c) 18 (d) 54

13. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is
(a) 10 (b)* 6 (c) 0 (d) 4

14. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is
(a)* $\frac{-1}{i+1}$ (b) $\frac{1}{i-1}$ (c) $\frac{-1}{i-1}$ (d) $\frac{1}{i+1}$

15. The real part of $\frac{1}{1-\cos\theta + i\sin\theta}$ is
(a) $\frac{1}{1-\cos\theta}$ (b)* $\frac{1}{2}$ (c) $\frac{\tan\theta}{2}$ (d) 2

16. The points representing complex number z for which $|z - 3| = |z - 5|$ lie on the locus given by
(a) circle (b) ellipse (c)* straight line (d) none of these

17. $|z - i| < |z + i|$ represents the region
(a) Re(z) > 0 (b) Re (z) < 0 (c)* Im (z) > 0 (d) Im (z) < 0

18. If x = 2 + 5i (where $i^2 = -1$), then the value of $(x^3 - 5x^2 + 33x - 29)$ is equal to
(a) 6 (b) 8 (c)* 0 (d) 12

19. If 1, ω, ω^2, ………., ω^{n-1} are n roots of unity, then $(1 - \omega)(1 - \omega^2)$ ……..$(1 - \omega^{n-1})$ is equal to
(a)* n (b) 1 (c) 0 (d) n^2

20. The maximum number of real of the equation $x^{2n} - 1 = 0$ is
(a)* 2 (b) 3 (c) n (d) none of these

21. If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then the locus of the point represented by z, is a
(a) circle (b)* straight line (c) parabola (d) none of these

22. The locus of the point z satisfying the condition arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ is
(a) parabola (b)* circle (c) pair of straight line (d) none

23. If $\frac{1-ix}{1+ix} = a + ib$, then $a^2 + b^2 =$
(a)* 1 (b) -1 (c) 0 (d) none of these

24. Let $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}, A = a + a^2 + a^4$ *and* $B = a^3 + a^5 + a^6$, then A and B are roots of the equation
(a) $x^2 - x + 2 = 0$ (b) $x^2 - x - 2 = 0$ (c)* $x^2 + x + 2 = 0$ (d) none of these

25. The complex number z = x + iy which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lies on
(a)* the axis of x (b) the straight line y = 5
(c) the circle passing through the origin (d) none of these

26. Let α and β be the roots of the equation $x^2 + x + 1 = 0$ the equation whose roots are α^{19}, β^7 is

(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$ (c) $x^2 + x - 1 = 0$ (d)* $x^2 + x + 1 = 0$

27. If the complex numbers iz, z and z + iz represent the three vertices of a triangle, then the area of the triangle is

(a)* $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{1}{2}|z-1|$ (d) $|z-1|^2$

28. The product of cube roots of -1 is equal to

(a)* -1 (b) 0 (c) -2 (d) 4

29. If magnitude of a complex number 4 – 3i is tripled and rotated by an angle π anticlockwise then resulting complex number would be

(a)* -12 + 9i (b) 12 + 9i (c) 7 – 6i (d) 7 + 6i

30. The complex number z_1, z_2, z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ then triangle is

(a)* equilateral (b) right angled triangle
(c) acute angled triangle (d) obtuse angled isosceles

31. If $|z - 2 - 3i| + |z + 2 - 6i| = 4$ where $i = \sqrt{-1}$ then locus of P(z) is

(a) an ellipse (b)* ϕ
(c) segment joining the point 2 + 3i, -2 + 6i (d) none

32. If $l_m\left(\frac{z-1}{2z+1}\right) = -4$, then locus of z is

(a)* ellipse (b) parabola (c) straight line (d) circle

33. If $\left|\frac{z+i}{z-i}\right| = 3$, then radius of the circle is

(a) $\frac{2}{\sqrt{21}}$ (b) $\frac{1}{\sqrt{21}}$ (c)* $\sqrt{3}$ (d) $\sqrt{21}$

34. If $\sqrt{3} + i = (a + ib)(c + id)$, then $\tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{d}{c}\right)$ has the value

(a) π/3 + 2nπ, n ∈ I (b)* nπ + π/6, n ∈ I

(c) nπ - π/3, n ∈ I (d) 2nπ - π/3 , n ∈ I

35. The amplitude of $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$ is

(a) $\frac{2\pi}{5}$ (b) $\frac{\pi}{15}$ (c)* $\frac{\pi}{10}$ (d) $\frac{\pi}{5}$

36. $\left|\frac{1}{2}(z_1+z_2)+\sqrt{z_1z_2}\right|+\left|\frac{1}{2}(z_1+z_2)-\sqrt{z_1z_2}\right|$ is equal to

(a) $|z_1+z_2|$ (b) $|z_1-z_2|$ (c)* $|z_1|+|z_2|$ (d) $|z_1|-|z_2|$

37. The value of $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ will be

(a) 1 (b)* -1 (c) 2 (d) -2

Equations, Inequations & Inequalities

1. The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) is

(a) three (b) two
(c) infinite (d)* no value of k will satisfy

2. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is

(a) $\frac{3abc + b^3}{a^3}$ (b) $\frac{a^3 + b^3}{3abc}$ (c)* $\frac{3abc - b^3}{a^3}$ (d) $\frac{-\left(3abc + b^3\right)}{a^3}$

3. If the ratio of the roots of the equation $x^2 + bx + c = 0$ is as that of $x^2 + qx + r = 0$, then

(a) $r^2b = qc^2$ (b) $r^2c = qb^2$ (c) $c^2r = q^2b$ (d)* $b^2r = q^2c$

4. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocals, then

(a)* ab^2, ca^2, bc^2 are in A.P (b) a^2b, c^2a, b^2c are in A.P
(c) ab^2, ca^2, bc^2 are in G.P (d) none of these

5. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2, 3 are roots of the equation f(x) = 0, the values of m and n are

(a) 5, -30 (b)* -5, 30 (c) 5, 30 (d) none of these

6. If α , β are the roots of $ax^2 + bx + c = 0$, and $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of

(a) $cx^2 + ax + b = 0$ (b) $bx^2 + ax + c = 0$
(c)* $cx^2 + bx + a = 0$ (d) $ax^2 + cx + b = 0$

7. If the equation $(\lambda - 1)x^2 + (\lambda + 1)x + (\lambda - 1) = 0$ has real roots , then λ can have any value in the interval

(a) $(1/3, 3)$ (b) $(-3, 3)$ (c) $(0, 3)$ (d)* $(1/3, 2)$

8. Let |x| and [x] denote the fractional integral part of a real number x respectively . Then solutions of 4|x| = x+[x] are

(a) $\pm\frac{2}{3}, 0$ (b) $\pm\frac{4}{3}, 0$ (c)* $0, \frac{5}{3}$ (d) ±2, 0

9. The value of 'a' for which the equations $x^3 + ax + 1 = 0$ and $x^4 + ax + 1 = 0$ have a common roots is

(a) 2 (b)* -2 (c) 0 (d) none of these

10. The roots of the equation $\left|x^2 - x - 6\right| = x + 2$ are

(a) $-2,1,4$ (b) $0,2,4$ (c) $0,1,4$ (d)* $-2,2,4$

11. If α and β are the roots of the equation $x^2 + px + 1 = 0$ and ν and δ are the root of $x^2 + qx + 1 = 0$, value of $(\alpha - \nu)(\beta - \nu)(\alpha + \delta)(\beta + \delta)$ is

(a) p^2-q^2 (b)* q^2-p^2 (c) p^2 (d) q^2

12. If α and β are the roots of the equation $ax^2 + bx + c = 0$ then $(1 + \alpha + \alpha^2)\ (1 + \beta^2 + \beta^2) =$

(a) 0 (b)* positive (c) negative (d) none of these

13. α, β be the root of $x^2 - 3x + a = 0$ and ν, δ are the root of the $x^2 - 12x + b = 0$ and numbers α, β, ν, δ (in order) from an increasing G.P., then

(a) a = 3, b = 12 (b) a = 12, b = 3 (c)* a = 2, b = 32 (d) a = 4, b = 16

14. The real roots of the equation $7^{\log_7\left(x^2 - 4x + 5\right)} = x - 1$

(a) 1 and 2 (b)* 2 and 3 (c) 3 and 4 (d) 4 and 5

15. The number of real solutions of $2\sin(e^x) = 5^x + 5^{-x}$ in [0, 1] is

(a)* 0 (b) 1 (c) 2 (d) 4

16. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is

(a) - 1/3 (b)* 2/3 (c) - 2/3 (d) 1/3

17. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is

(a) 3 (b) 2 (c)* 4 (d) 1

18. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

(a) $x^2 - 18x - 16 = 0$ (b)* $x^2 - 18x + 16 = 0$

(c) $x^2 + 18x - 16 = 0$ (d) $x^2 + 18x + 16 = 0$

19. If (1 – p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are

(a) -1, 2 (b) -1, 1 (c)* 0, -1 (d) 0, 1

20. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is

(a) 4 (b) 12 (c) 3 (d)* 49/4

21. The value of α for which the sum of the squares of the roots of the equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$ assume the least value is

(a)* 1 (b) 0 (c) 3 (d) 2

22. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals

(a) -2 (b) 3 (c) 2 (d)* 1

23. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(a) (5, 6] (b) (6, ∞) (c)* (-∞, 4) (d) [4, 5]

24. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval

(a)* $-1 < m < 3$ (b) $1 < m < 4$ (c) $-2 < m < 0$ (d) $m > 3$

25. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15° respectively then the value of $2 + q - p$ is

(a) 0 (b) 1 (c) 2 (d)* 3

26. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p +q) is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c)* $\sqrt{2}$ (d) 2

27 A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1,3] is

(a) $\frac{1}{2}\log_e 3$ (b) $\log_3 e$ (c) $\log_e 3$ (d)* $2\log_3 e$

28. Let A (h, k), B (1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

(a) {0 , 2} (b)* {-1 , 3} (c) {-3 , -2} (d) {1 , 3}

29. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less then $\sqrt{5}$, then the set of possible values of a is

(a) $(-3, \infty)$ (b) $(3, \infty)$ (c) $(-\infty, -3)$ (d)* (- 3 , 3)

30. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 0 (b)* 1 (c) 2 (d) -1

Punjab Edusat Society

PROBLEM SET ON PROBABILITY

1. Two fair dice are tossed. Let X be the event that the first die show an even number and Y be the event that second die show an odd number. The two events X and Y are

(a) mutually exclusive (b) independent and mutually exclusive

(c) dependent (d)* none of these

2. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

(a)* 0.39 (b) 0.25 (c) 0.11 (d) none of these

3. For a biased die the probabilities for the different faces to turn up are given below:

Face	1	2	3	4	5	6
Probability	0.1	0.32	0.21	0.15	0.05	0.17

This die is tossed and you are told that either face 1 or face 2 has found up. Then the probability that it is face 1 is

(a)* $\frac{5}{21}$ (b) $\frac{8}{21}$ (c) $\frac{11}{21}$ (d) $\frac{13}{21}$

4. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of determinant chosen is positive, is

(a)* $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{7}{16}$ (d) $\frac{9}{16}$

5. If A and B are two events such that P(A) > 0, and P(B) ≠ 1, the $P\left(\bar{A}/\bar{B}\right)$ is equal to

(a) 1 – P(A/B) (b) $1-\left(\bar{A}/B\right)$ (c)* $\frac{1-P\left(A\cup B\right)}{P\left(\bar{B}\right)}$ (d) $\frac{P\left(\bar{A}\right)}{P\left(\bar{B}\right)}$

6. Fifteen coupons are numbered 1, 2, ……….., 15 respectively. Seven coupons are selected at random time with replacement. The chance that the largest number appearing on a selected coupon is 9, is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$ (c) $\left(\frac{3}{5}\right)^7$ (d)* none of these

7. If the letters of word "ASSASSIN" are written down at random in a row, the probability that no two S's occur together, is

(a)* 1/14 (b) 1/35 (c) 1/7 (d) none of these

8. Three identical dice are rolled. The probability that the same number will appear on each of them is

(a) 1/6 (b)* 1/36 (c) 1/18 (d) 3/28

9. If M and N are any two events, the probability that exactly one of them occurs is

(a)* $P(M) + P(N) - 2P(M \cap N)$ (b) $P(M) + P(N) - P(M \cap N)$

(c)* $P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$ (d) $P(M \cap \bar{N}) - (\bar{M} \cap N)$

10. A box contain 100 tickets numbered 1, 2, ……………, 100. Two tickets are chosen at random. It is given that the maximum number on the tickets is not more than 10. Then the probability that minimum on them is 5, is

(a)* 1/9 (b) 2/9 (c) 1/3 (d) 2/5

11. $P(A \cup B) = P(A \cap B)$ is possible only when

(a)* $P(A) = P(B)$ (b) $P(A) > P(B)$

(c) $P(A) < P(B)$ (d) none of these

12. If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then p lies in the interval

(a) $0 \leq p < ½$ (b)* $1/3 \leq p \leq ½$ (c) $0 \leq p \leq 1/3$ (d) none of these

13. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or test I and III. The probability of the student passing in tests I, II and III are p, q and ½ respectively. If the probability that the student is successful is ½, then

(a) p = q = 1 (b) p = q = ½ (c)* p = 1, q = 0 (d) p = 1, q = ½

(e) none of these

14. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with the probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8 (c)* 1.2 (d) 1.4

15.** For two given events A and B, $P(A \cap B)$ is

(a)* Not less than $P(A) + P(B) - 1$ (b)* Not greater than $P(A) + P(B)$

(c)* Equal to $P(A) + P(B) - P(A \cup B)$ (d) Equal to $P(A) + P(B) + P(A \cup B)$

16. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one ball is now drawn at random from urn A, the probability that it is found out be red is

(a)* 32/55 (b) 41/55 (c) 43/55 (d) ½

17. In three throws of two dice, the probability of throwing doublets not more than twice is

(a) 1/6 (b) 5/72 (c)* 215/216 (d) none

18. Given a throw of three unbiased dice show different faces, the probability that at least one face shows 6 is

(a) 5/6 (b) 5/18 (c)* ½ (d) 13/18

19. Bag A contains 2 white and 2 red balls and another bag B contains 4 white and 5 red balls. A ball is drawn and is found to be red. The probability that it was drawn from the bag B is

(a) 25/52 (b) ½ (c)* 10/19 (d) 5/19

20. Let A and B be two events such that P(A) = 7/20, P(B) = 9/20, P(A ∪ B) = 11/20, $P\left(A \cup \bar{A}\right) = 1,$ then the value of $P\left(\bar{A} \cup \bar{B}\right)$ is equal to

(a) ¼ (b)* ¾ (c) 1/10 (d) none of these

21. A bag contains 5 brown and 4 white socks. A man pulls out two socks without replacement. The probability that they are of the same colour is

(a) 5/108 (b) 1/6 (c) 5/18 (d)* 4/9

22. A six faced die is so biased that it is twice to show an even number as an odd number when thrown. If it is thrown twice the probability that the sum of two numbers thrown is even, is

(a) 4/9 (b)* 5/9 (c) 1/9 (d) none of these

23. In shuffling a pack of playing cards, four cards accidently dropped. The probability that the missing cards should be one from each suit is

(a) $\frac{1}{256}$ (b) $\frac{4}{20825}$ (c)* $\frac{2197}{20825}$ (d) none of these

24. The probability that a student is not a swimmer is 1/5. What is the probability that out of 5 students, 4 are swimmer?

(a)* $^5C_4\left(\frac{4}{5}\right)^4\frac{1}{5}$ (b) $\left(\frac{4}{5}\right)^4\frac{1}{5}$ (c) $^3C_1\frac{1}{5}\left(\frac{4}{5}\right)^4\times{}^5C_4$ (d) none

25. four persons are selected at random from a group of 3 men, 2 women and 4 children. What is the chance that exactly 2 of them are children?

(a) 9/21 (b) 10/23 (c) 11/24 (d)* 10/21

26. A person writes 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, what is the probability that all letters are not placed in the right envelopes?

(a) 1/24 (b) 11/24 (c) 15/24 (d)* 23/24

27. A coin is tossed 3 times by 2 persons. What is the chance that both get equal number of heads?

(a) 3/8 (b) 1/9 (c)* 5/16 (d) none of these

28. Out of 15 tickets marked with the numbers from 1 to 15, three are drawn at random. What is the chance that the numbers on them are in A.P?

(a)* 7/65 (b) 9/15 (c) 13/261 (d) none of these

29. The probability of event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B occurring is

(a) 0.6 (b) 0.5 (c) 0.7 (d)* none of these

30. If A and B are two independent events, the probability that only one of A or B occur is

(a)* $P(A) + P(B) - 2P(A \cap B)$ (b) $P(A) + P(B) - P(A \cap B)$
(c) $P(A) + P(B)$ (d) none of these

31. If A and B are two independent events, the probability that both A and B occur is 1/8 and the probability that neither of them occurs 3/8. The probability of the occurrence of A is

(a)* ½ (b) 1/3 (c) ¾ (d) 1/5

32. Twelve balls are distributed among three boxes. The probability that the first box contains 3 ball is

(a)* $\frac{110}{9}\left(\frac{2}{3}\right)^{10}$ (b) $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$ (c) $\frac{^{12}C_3}{12^3}.2^9$ (d) $\frac{^{12}C_3}{3^{12}}$

33. A fair coin is tossed a fix number of times . If the probability of getting 4 heads equal the probability of getting 7 heads .Then the probability of getting 2 heads is

(a)* $\frac{35}{2048}$ (b) $\frac{3}{4096}$ (c) $\frac{1}{1024}$ (d) none of these

34. If A and B are two events that the value of the determinant chosen at random from all the determinants of order 2 with entries 0 or 1 only is positive or negative respectively . Then

(a) $P(A) > P(B)$ (b)* $P(A) \le P(B)$ (c) $P(A) = P(B) = 1/2$ (d) none of these

35. The probability of A = probability of B = probability of $C = \frac{1}{4}, P(A \cap B \cap C) = 0$

$P(B \cap C) = 0, P(A \cap C) = \frac{1}{8},$ and $P(A \cap B) = 0$, the probability that at least one of the events $A, B, or\ C$ will occur, is

(a)* $\frac{5}{8}$ (b) $\frac{37}{64}$ (c) $\frac{3}{4}$ (d) 1

36. India plays two matches each with West indies and Australia . In any match the probabilities of India getting points 0 , 1 and 2 are 0.45 , 0.05 respectively . Assuming that the outcomes are Independent. The probability of India getting at least 7 points is .

(a) 1/7 (b) .0675 (c)* .0875 (d) none of these

37. X is random variable with probability function $P(x) = {}^3C_x \left(\frac{1}{2}\right)^3, x = 0,1,2,3$ the value of $E(X^2 + 2)$ is

(a) 7 (b) 17/4 (c)* 5 (d) 4

38. If $P(A \cap B) = \frac{1}{3}, P(A \cup B) = \frac{5}{6}$ and $P(A) = \frac{1}{2}$, then which one of the following is correct?

(a)* $A\ and\ B$ are independent events (b) $A\ and\ B$ are mutually exclusive events

(c) $P(A) = P(B)$ (d) $P(A) < P(B)$

39. Bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted it is green . The probability that it comes from bag B

(a) 2/7 (b) 2/3 (c)* 3/7 (d) 1/3

40. For a party 8 guests are invited by a husband and his wife . They sit around a circular table
for dinner . the probability that the husband and his wife sit together is :

(a) $\frac{2}{7}$ (b)* $\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

41. One hundred identical coins, each with probability p of showing heads are tossed once .
If $0 < p < 1$ and the probability of head showing on 50 coins is equal to that of head showing on 51 coins, the value of p is:

(a) $\frac{1}{2}$ (b)* $\frac{51}{101}$ (c) $\frac{49}{101}$ (d) none of these

42. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first , second , third and fourth shot are 0.4 , 0.3,0.2, and 0.1 respectively. What is the probability that the gun hits the plane

(a) 0.6977 (b) 0.6967 (c)* 0.6976 (d) none of these

43. Five horses are in a race . Mr A selects two of the horse at random and bets on them, the probability that Mr A selected the winning horse is

(a)* 2/5 (b) 4/5 (c) 3/5 (d) 1/5

44. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$

The probability that they contradict each other when asked to

(a) 4/5 (b) 1/5 (c)* 7/20 (d) 3/20

45. A random variables X has the probability distribution:

X:	1	2	3	4	5	6	7	8
P(X):	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X$ is a prime number$\}$ and $F = \{X < 4\}$, the probability $P(E \cup F)$ is :

(a) 0.50 (b) *0.77 (c) 0.35 (d) 0.87

46. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

(a)* $\frac{28}{256}$ (b) $\frac{219}{256}$ (c) $\frac{128}{256}$ (d) $\frac{37}{256}$

47. Three houses are available in a locality .Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is.

(a) $\frac{2}{9}$ (b)* $\frac{1}{9}$ (c) $\frac{8}{9}$ (d) $\frac{7}{9}$

48. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals

(a) $\frac{2}{e^2}$ (b) 0 (c)* $1 - \frac{3}{e^2}$ (d) $\frac{3}{e^2}$

49. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are

(a) equally likely and mutually exclusive (b) equally likely but not independent

(c)* independent but not equally likely (d) mutually exclusive and independent.

50. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is

(a) $\frac{6}{55}$ (b)* $\frac{6}{e^5}$ (c) $\frac{6}{5^e}$ (d) None of these

Punjab Edusat Society

TRIGONOMETRY

1. If $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$, then x =

(a) ½ (b) 1/3 (c) 1/6 (d) 1/10

2. The value of $\sin(\cot^{-1} x)$ =

(a) $\sqrt{1+x^2}$ (b) x (c) $(1+x^2)^{-3/2}$ (d) $(1+x^2)^{-1/2}$

3. tan 10° + tan 35° + tan 10° . tan 35° =

(a) 0 (b) ½ (c) -1 (d) 1

4. The general value of θ satisfying the equation sin θ = - ½ and tan θ = $1/\sqrt{3}$ is

(a) $n\pi + \frac{\pi}{6}, n \in I$ (b) $n\pi + (-1)^n\left(\frac{7\pi}{6}\right), n \in I$

(c) $2n\pi + \frac{7\pi}{6}, n \in I$ (d) $2n\pi + \frac{11\pi}{6}, n \in I$

5. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}, -\frac{\pi}{2} < A < 0, -\frac{\pi}{2} < B < 0$, the value of 2 sin A + 4 sin B is equal to

(a) 4 (b) 2 (c) -4 (d) 0

6. From the top of a light house the angle of depression of a boat is 15. If the light house is 60 m high and its base is sea-level, the distance of the boat from the light house is

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}60$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}60$ (c) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 60$ (d) none of these

7. The value of $\log_3 \tan 1° + \log_3 \tan 2° + \ldots\ldots\ldots\ldots + \log_3 \tan 89°$ is

(a) 3 (b) 1 (c) 2 (d) 0

8. If in a triangle ABC, c = 2a cos B, then the triangle is a/an

(a) simple triangle (b) isosceles triangle

(c) equilateral triangle (d) right-angle triangle

9. AB is a vertical diameter of circle and PQ is a diameter at an angle θ to AB. The value of θ, so that the time of sliding down PQ any be $\sqrt{2}$ times that of sliding down AB, is

(a) π/6 (b) π/4 (c) π/3 (d) none of these

10. A flagstaff, 10 meters high, stands at the centre of an equilateral triangle, which is horizontal. On the top of the flagstaff each side subtends an angle of 60°. The length of the each side is

(a) $6\sqrt{3}$ (b) $4\sqrt{6}$ (c) $5\sqrt{6}$ (d) $6\sqrt{5}$

11. If the area of a triangle ABC is $a^2 - (b - c)^2$, then tan A equals

(a) - 8/15 (b) 15/16 (c) 8/15 (d) 15/8

12. If $\cot^{-1}\left(\sqrt{\cos\alpha}\right) + \tan^{-1}\left(\sqrt{\cos\alpha}\right) = u$, then sin u equals

(a) $\tan^2 \alpha$ (b) $\tan 2\alpha$ (c) 1 (d) $\cot^2 \alpha/2$

13. If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$. Then $9x^2 - 12xy \cos \theta + 4y^2$ is equal to

(a) 36 (b) $-36 \sin^2 \theta$ (c) $36 \sin^2 \theta$ (d) $36 \cos^2 \theta$

14. If $\cos \alpha + \cos \beta = a$, $\sin \alpha + \sin \beta = b$ then $\cos(\alpha - \beta)$ is equal to

(a) $\frac{2ab}{a^2 + b^2}$ (b) $\frac{a^2 + b^2}{a^2 - b^2}$ (c) $\frac{a^2 + b^2 - 2}{2}$ (d) $\frac{b^2 - a^2}{a^2 + b^2}$

15. If cos A = ¾, then value of $32 \sin\frac{A}{2}\sin\frac{5A}{2}$ is

(a) $\sqrt{11}$ (b) $-\sqrt{11}$ (c) 11 (d) -11

16. If $\tan\theta \cdot \tan\left(\frac{\pi}{3} + \theta\right) \cdot \left(\frac{-\pi}{3} + \theta\right) = k \tan 3\theta$, then the value of k is

(a) 1 (b) 1/3 (c) -1 (d) none of these

17. The value of $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$ is

(a) 1 (b) -1 (c) ½ (d) -3/2

18. If $\cos(\theta + \phi) = m \cos(\theta - \phi)$, then $\tan \theta =$

(a) $[(1 + m)/(1 - m)] \tan \phi$ (b) $[(1 - m)/(1 + m)] \tan \phi$

(c) $[(1 - m)/(1 + m)] \cot \phi$ (d) $[(1 + m)/(1 - m)] \sec \phi$

19. If cos 20° = k and $\cos x = 2k^2 - 1$, then the possible values of x between 0° and 360° are

(a) 140° (b) 40° and 140° (c) 40° and 320° (d) 50° and 130°

20. The equation a sin x + b cos x = c where $|c| > \sqrt{a^2 + b^2}$ has

(a) a unique solution (b) infinite number of solutions

(c) no solution (d) none of these

21. A tree is broken by wind, its upper part touches the ground at a point 10 metres from the foot of the tree and makes an angles of 60° with the ground the entire length of the tree is

(a) 15 m (b) 20 m (c) $10\left(1+\sqrt{2}\right)$ (d) $10\left(1+\frac{\sqrt{3}}{2}\right)m$

22. If the angles of triangle are 30° and 45° and the included side is $\left(\sqrt{3}+1\right)$ cms, then the area of triangle is

(a) $\frac{1}{\sqrt{3}-1}$ (b) $\sqrt{3}+1$ (c) $\frac{1}{\sqrt{3}+1}$ (d) none of these

23. $\tan^{-1}\left(\frac{1}{11}\right)+\tan^{-1}\left(\frac{2}{12}\right)=$

(a) $\tan^{-1}\left(\frac{33}{132}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$ (c) $\tan^{-1}\left(\frac{132}{33}\right)$ (d) none of these

24. If sin A = sin B and cos A = cos B, then A =

(a) $2n\pi + B$ (b) $2n\pi - B$ (c) $n\pi + B$

(d) $n\pi + (-1)^n B$

25. If the perimeter of a triangle ABC is 6 times the A.M.'s of the sine of its angles and the side a is 1, then angle A is

(a) 30° (b) 45° (c) 60° (d) 120°

26. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by (n integer)

(a) $x = 2n\pi$ (b) $x = 2n\pi + \frac{\pi}{2}$

(c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$ (d) none of these

27. In a ΔABC A : B : C = 3 : 5 : 4. Then a + b + $c\sqrt{2}$ is equal to

(a) 2b (b) 2c (c) 3b (d) 3a

28. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is

(a) π/3 (b) π/2 (c) 2π/3 (d) 5π/6

29. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is equal to

(a) 1/64 (b) 1/32 (c) 1/16 (d) 1/8

30. The value of sin 10° + sin 20° + sin 30° + ………….. + sin 360° is

(a) 1 (b) 0 (c) -1 (d) none of these

31. If x + 1/x = 2, the principle value of $\sin^{-1} x$ is

(a) π/4 (b) π/2 (c) π (d) 3π/2

32. The smallest angle of the triangle whose sides are $6+\sqrt{12}, \sqrt{48}, \sqrt{24}$ is

(a) π/3 (b) π/4 (c) π/6 (d) none of these

33. Given $A = \sin^2\theta + \cos^4\theta$, then for all real values of θ

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$ (c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

34. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, ...C_n$ are constants and $C_n \ne 0$. Then the value of n is

(a) 6 (b) 3 (c) 2 (d) 1

35. The area of ΔABC is

(a) $\frac{s(s-a)(s-b)(s-c)}{2}$ (b) $\frac{b^2 \sin C \sin A}{\sin B}$

(c) ab sin C (d) $\frac{1}{2}\frac{a^2 \sin B \sin C}{\sin A}$

36. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

37. The x satisfying $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$ are

(a) 0 (b) 1, -1 (c) 0, ½ (d) none of these

38. If the radius of the circum circle of an isosceles triangle PQR is equal to PQ or PR (PQ = PR)

Then the angle P must be

(a) π/6 (b) π/3 (c) π/2 (d) 2π/3

39. Which is correct one?
(a) sin 1° > sin 1 (b) sin 1° = sin 1 (c) sin 1° = sin π/180 (d) None of these

40. The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots\ldots\ldots + \sin^2 90°$ is
(a) 9.5 (b) 9 (c) 10 (d) 8

41. The value of the expression $\sin^6 \theta + \cos^6 \theta + 3\sin^2 \theta\cos^2 \theta$
(a) is 0 (b) is 2 (c) greater than 3 (d) is -1

42. The value of the expression cos 1° cos 2° ……….. cos 179° equals
(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) -1

43. If α + β = π/2 and β + γ = α, then tan α equals
(a) 2(tan β + tan γ) (b) tan β + tan γ (c) tan β + 2tan γ (d) 2tan β + tan γ

44. The number of solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is
(a) zero (b) one (c) two (d) infinite

45. $y = \sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2}$, then the value of $\frac{dy}{dx}$ is :
(a) 1 (b) -1 (c) 0 (d) 2

46. On the bank of river there is a tree. On another bank, an observer makes an angle of elevation of $60°$ at a top of the tree . The angle of elevation of the top of the tree at a distance 20m away from the bank is $30°$ .The width of the river is:
(a) 20m (b) 10m (c) 5m (d) 1m

47. If $\tan^{-1}\frac{x-1}{x+1} = \frac{1}{2}\tan^{-1} x$, then value of x is:
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 2

48. Points D,E are taken on the side BC of the triangle ABC, such that $BD = DE = EC$.
If $\angle BAD = x$, $\angle DAE = y, \angle EAC = z$, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is equal to:
(a) 1 (b) 2 (c) 4 (d) none of these

49. If p is the product of the sines of angles of a triangle and q be the product of their cosines, then tangents of the angle are roots of the equation :

(a) $qx^3 - px^3 + (1+q)x - p = 0$ (b) $qx^3 - qx^2 + (1+p)x - q = 0$

(c) $(1+q)x^3 - px^2 + qx - p = 0$ (d) none of these

50. All the values of x for which expression $(1+\tan x + \tan^2 x)(1 - \cot x + \cot^2 x)$ is positive satisfy

(a) $0 \le x \le \frac{\pi}{2}$ (b) $0 \le x \le \pi$ (c) for all $x \in R$ (d) $x = 0$

51. The equation $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has:

(a) no solution (b) unique solution

(c) infinite number of solutions (d) none of these

52. The angle of a elevation of the top of the tower observed from each of the three points $A, B,$ Cone the ground forming a triangle is the same angle α. If R is the circum-radius of the triangle ABC, then the height of the tower is:

(a) $R\sin\alpha$ (b) $R\cos\alpha$ (c) $R\cot\alpha$ (d) $R\tan\alpha$

53. In a triangle $\Delta ABC, a, c, A$ are given and b_1, b_2 are two values, of the third side b is, such that $b_2 = 2b_1$ then $\sin A$ is equal to :

(a) $\frac{\sqrt{9a^2 - c^2}}{8a^2}$ (b) $\frac{\sqrt{9a^2 - c^2}}{8a^2}$ (c) $\frac{\sqrt{9a^2 + c^2}}{8a^2}$ (d) none of these

54. The angle of elevation of top of a tower from a point on the ground is 30° and it is 60° when it is viewed from a point located 40 m away from the initial point towards the tower. The height of the tower is

(a) $-20\sqrt{3}m$ (b) $\frac{\sqrt{3}}{20}m$

(c) $-\frac{\sqrt{3}}{20}m$ (d) $20\sqrt{3}m$

55. In ΔABC, ∠A = π/2, b = 4, c = 3, then the value of R/r is equal to

(a) 5/2 (b) 7/2 (c) 9/2 (d) 35/24

56. If a^2, b^2, c^2 are in A.P, then which of the following is also in A.P?

(a) sin A, sin B, sin C (b) tan A, tan B, tan C

(c) cot A, cot B, cot C (d) none of these

57. The solution set of the equation $\sin^{-1} x = 2\tan^{-1} x$ is:

(a) {1, 2} (b) {-1, 2} (c) {-1, 1, 0} (d) {1, ½, 0}

58. Let $\cos(2\tan^{-1} x) = ½$, then the value of x is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $1-\sqrt{3}$ (d) $1-\frac{1}{\sqrt{3}}$

59. If $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=\tan^{-1}\frac{2x}{1-x^2}$, then the value of x is

(a) a (b) b (c) $\frac{a+b}{1-ab}$ (d) $\frac{a-b}{1+ab}$

60. If cos θ = cos α cos β, then $\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)$ is equal to

(a) $\tan^2\frac{\alpha}{2}$ (b) $\tan^2\frac{\beta}{2}$ (c) $\tan^2\frac{\theta}{2}$ (d) $\cot^2\frac{\beta}{2}$

61. A house subtends a right angle at the window of an opposite house and the angle of elevation of the window from the bottom of the first house is $60°$. If the distance between the two houses be 6m, then the height of the first house is

(a) $8\sqrt{3}\,m$ (b) $6\sqrt{3}\,m$ (c) $4\sqrt{3}\,m$ (d) None of these

62. If $\cos\theta=\frac{8}{17}$ and θ lies in the Ist quadrant, then the value of $\cos(30°+\theta)+\cos(45°-\theta)+\cos(120°-\theta)$

(a) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$ (b) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2}+\frac{1}{\sqrt{2}}\right)$

(c) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}-\frac{1}{\sqrt{2}}\right)$ (d) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2}-\frac{1}{\sqrt{2}}\right)$

63. The root of the equation $1-\cos\theta=\sin\theta.\sin\frac{\theta}{2}$ is

(a) $k\pi, k\in I$ (b) $2k\pi, k\in I$ (c) $k\frac{\pi}{2}, k\in I$ (d) None of these.

64. In ΔABC, if $\sin^2\frac{A}{2},\sin^2\frac{C}{2}$ be in H.P. then a,b,c will be in

(a) AP (b) GP (c) HP (d) None of these

65. In any triangle ABC, $\frac{\frac{A}{2}-\tan\frac{B}{2}}{\tan\frac{A}{2}+\tan\frac{B}{2}}$ is equal to

(a) $\frac{a-b}{a+b}$ (b) $\frac{a-b}{c}$ (c) $\frac{a-b}{a+b+c}$ (d) $\frac{c}{a+b}$

66. The sides of triangle are $3x+4y, 4x+3y$ and $5x+5y$ where $x, y>0$ then the triangle is

(a) right angle (b) obtuse angled (c) equilateral (d) none of these

67. In a triangle with sides a, b, c, $r_1>r_2>r_3$ (which are the ex-radii) then

(a) $a>b>c$ (b) $a<b<c$ (c) $a>b$ and $b<c$ (d) $a<b$ and $b>c$

68. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a , is

(a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{n}\right)$ (c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{2n}\right)$

69. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \pi/6$ and $\angle ABE = \pi/3$, then the area of the ΔABC is

(a) 64/3 (b) 8/3 (c) 16/3 (d) $\frac{32}{3/\sqrt{3}}$

70. If in a triangle ABC $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$, then the sides a, b and c

(a) satisfy a + b = c (b) are in A.P (c) are in G.P (d) are in H.P

71. If $\pi < \alpha - \beta < 3\pi$, $\sin\alpha + \sin\beta = -21/65$ and $\cos\alpha + \cos\beta = -27/65$, then $\cos\frac{\alpha-\beta}{2}$ is

(a) - 6/65 (b) $\frac{3}{\sqrt{130}}$ (c) 6/65 (d) $-\frac{3}{\sqrt{130}}$

72. If $u=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$, then the difference between the maximum and minimum value of u^2 is given by

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$ (c) $(a+b)^2$ (d) $2(a^2+b^2)$

73. The sides of a triangle are sin α, cos α and $\sqrt{1+\sin\alpha\cos\alpha}$ for some 0 < α < π/2. Then the greatest angle of the triangle is

(a) 150° (b) 90° (c) 120° (d) 60°

74. A person standing on the bank of river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breath of the river is

(a) 60 m (b) 30 m (c) 40 m (d) 20 m

75. If $\cos^{-1} x - \cos^{-1} y/2 = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to

(a) 2sin 2α (b) 4 (c) $4\sin^2 \alpha$ (d) $-4\sin^2 \alpha$

76. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then

sin A. Sin B. sin C are in

(a) G.P (b) A.P (c) A.G.P (d) H.P.

77. In triangle PQR, ∠R = π/2. If tan P/2 and tan Q/2 are roots of the equations $ax^2 + bx + c = 0$ (a≠0) then

(a) a + b = c (b) b + c = a (c) a + c = b (d) b = c

78. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of triangle, then

2(r + R) is equal to

(a) a + b (b) b + c (c) c + a (d) a + b – c

79. If 0 < x < π, and cos x + sin x = ½, then tan x is

(a) $-(4+\sqrt{7})/3$ (b) $(1+\sqrt{7})/4$ (c) $(1-\sqrt{7})/4$ (d) $(4-\sqrt{7})/3$

80. The number of values of x in the interval [0, 3π] satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 6

81. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is

(a) 3 (b) 4 (c) 5 (d) 1

82. A tower stands at the centre of a circular park .A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower , and the angle of elevation of the top of the tower from A or B is $30°$. The height of the tower is

(a) $2a\sqrt{3}$ (b) $a/\sqrt{3}$ (c) $a\sqrt{3}$ (d) $2a/\sqrt{3}$

83. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{4}{17}$ (b) $\frac{5}{17}$ (c) $\frac{6}{17}$ (d) $\frac{3}{17}$

84. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is $60°$. He moves away from the pole along the line BC to a point D such that CD = 7m. From D the angle of elevation of the point A is $45°$. Then the height of the pole is

(a) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (b) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$ (c) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (d) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

Answers

1.	c	**2.**	d	**3.**	d	**4.**	c	**5.**	c	**6.**	b
7.	d	**8.**	b	**9.**	c	**10.**	c	**11.**	c	**12.**	c
13.	c	**14.**	c	**15.**	c	**16.**	c	**17.**	d	**18.**	c
19.	c	**20.**	c	**21.**	c	**22.**	a	**23.**	d	**24.**	c
25.	a	**26.**	c	**27.**	c	**28.**	c	**29.**	a	**30.**	b
31.	b	**32.**	c	**33.**	b	**34.**	a	**35.**	d	**36.**	c
37.	c	**38.**	d	**39.**	c	**40.**	a	**41.**	d	**42.**	a
43.	c	**44.**	c	**45.**	c	**46.**	b	**47.**	b	**48.**	c
49.	a	**50.**	c	**51.**	b	**52.**	d	**53.**	b	**54.**	c
55.	a	**56**	**c**	**57.**	**c**	**58**	**b**	**59**	**d**	**60**	**b**
61.	**a**	**62.**	**a**	**63.**	**b**	**64.**	**c**	**65.**	b		
66.	b	**67.**	a	**68.**	c	**69.**	d	**70.**	b	**71.**	d
72.	a	**73.**	c	**74.**	**d**	**75.**	c	**76.**	b	**77.**	a
78.	a	**79.**	a	**80.**	**c**	**81.**	a	**82.**	b	**83.**	c
84.	d										

MATHEMATICS

PROBLEM SET ON VECTORS & 3D

1. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ *and* $3\hat{i}+\hat{j}-\hat{k}$ to the point $5\hat{i}+4\hat{j}-\hat{k}$. The total work done by the forces is

(a) 50 units (b) 20 units (c) 30 units
(d)* 40 units

2. The vectors $\vec{AB}=3\hat{i}+4\hat{k}$ *and* $\vec{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d)* $\sqrt{33}$

3. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k},\ \hat{i}-6\hat{j}+10\hat{k},\ -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then quadrilateral ABCD is a

(a)* Parallelogram but not a rhombus (b) Square
(c) Rhombus (d) Rectangle

4. Let $\vec{a},\vec{b}$ *and* $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ collinear with $\vec{c}$ *and* $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals

(a) 0 (b) $\lambda\vec{b}$ (c)* $\lambda\vec{c}$ (d) $\lambda\,\vec{a}$

5. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ *and* $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by

(a) 15 (b) 30 (c) 25 (d)* 40

6. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, then the vectors $\vec{a}+\vec{b}+3\vec{c},\ \lambda\vec{b}+4\vec{c}$ *and* $(2\lambda-1)\vec{c}$ are non-coplanar for

(a) no value of λ (b) all except one value of λ
(c)* all except two values of λ (d) all values of λ

7. Let $\vec{u},\vec{v},\vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection $\vec{v}$ *along* $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ *and* $\vec{v},\vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals

(a) 14 (b) $\sqrt{7}$ (c)* $\sqrt{14}$ (d) 2

8. Let $\vec{a},\vec{b}$ *and* $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ *and* $\vec{c}$, then sin θ equals

(a)* $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

9. If C is the mid point of AB and P is any point outside AB, then

(a)* $\vec{PA}+\vec{PB}=2\vec{PC}$ (b) $\vec{PA}+\vec{PB}=\vec{PC}$
(c) $\vec{PA}+\vec{PB}+2\vec{PC}=\vec{0}$ (d) $\vec{PA}+\vec{PB}+\vec{PC}=\vec{0}$

10. For any vector $\vec{a}$, the value of $\left(\vec{a}\times\hat{i}\right)^2+\left(\vec{a}\times\hat{j}\right)^2+\left(\vec{a}\times\hat{k}\right)^2$ is equal to

(a) $3\vec{a}^2$ (b) $\vec{a}^2$ (c)* $2\vec{a}^2$ (d) $4\vec{a}^2$

11. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k},\ \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is

(a)* the Geometric Mean of **a** and **b** (b) the arithmetic Mean of **a** and **b**

(c) equal to zero (d) the Harmonic mean of **a** and **b**

12. If $\vec{a},\vec{b},\vec{c}$ are non coplanar vectors and λ is a real number then $\left[\lambda\left(\vec{a}+\vec{b}\right)\ \ \lambda^2\vec{b}\ \ \lambda\vec{c}\right]=\left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}\right]$ for

(a) exactly one value of λ (b)* no value of λ

(c) exactly three values of λ (d) exactly two values of λ

13. Let $\vec{a}=\hat{i}-\hat{k},\ \vec{b}=x\hat{i}+y\hat{j}+(1-x)\hat{k},\ \vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$, then $\left[\vec{a},\vec{b},\vec{c}\right]$ depends upon

(a) only x (b) only y (c)* neither x or y (d) both x and y

14. The two lines x = ay + b, z = cy + d; and x = a′y + b′, z = c′y + d′ are perpendicular to each other if

(a) $\frac{a}{a'}+\frac{c}{c'}=-1$ (b) $\frac{a}{a'}+\frac{c}{c'}=1$ (c)* aa′ + cc′ = -1 (d) aa′ + cc′ = 1

15. The values of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k},\ \hat{i}-3\hat{j}-5\hat{k}\ and\ a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right-angled triangle with C = π/2 are

(a) -2 and 1 (b) 2 and -1 (c)* 2 and 1 (d) -2 and -1

16. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them , then $2\hat{u}\times 2\hat{v}$ is a unit vec tor for

(a) More than two values of θ (b) No value of θ

(c)* exactly one value of θ (d) exactly two values of θ

17. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k},\ \bar{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\bar{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals

(a) 1 (b) - 4 (c)* - 2 (d) 0

18. Let L be the line of intersection of the planes 2x + 3y + z = 1 and x + 3y + 2z = 2. If L makes an angle α with the positive x-axis, then cos α equals

(a) ½ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d)* $\frac{1}{\sqrt{3}}$

19. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

(a) π/3 (b) π/4 (c)* π/2 (d) π/6

20. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them , then $2\hat{u}\times 2\hat{v}$ is a unit vector for

(a) More than two values of θ (b) No value of θ
(c)* exactly one value of θ (d) exactly two values of θ

21. If $(2,3,5)$ is one end of a diameter of the sphere $x^2+y^2+z^2-6x-12y-2z+20=0$, then the coordinates of the other end of the diameter are

(a) (4 , -3 , 3) (b) (4 , 3 , 5) (c) (4 , 3 , -3) (d)* (4 , 9 , -3)

22. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k}$, $\bar{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\bar{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals

(a) 1 (b) - 4 (c)* - 2 (d) 0

23. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(a) $\alpha=2,\beta=1$ (b)* $\alpha=1,\beta=1$
(c) $\alpha=2,\beta=2$ (d) $\alpha=1,\beta=2$

24. The non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ are related by and $\vec{c}=-7\vec{b}$. then the angle between $\vec{a}$ and $\vec{c}$ is

(a) $\frac{\pi}{2}$ (b)* π (c) 0 (d) $\frac{\pi}{4}$

25. The line passing through the points $(5,1,a)$ and $(3,b,1)$ crosses the yz-plane at the point $\left(0,\frac{17}{2},\frac{-13}{2}\right)$. Then

(a)* $a=6,b=4$ (b) $a=8,b=2$ (c) $a=2,b=8$ (d) $a=4,b=6$

26. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(a) 2 (b) −2 (c)* -5 (d) 5

27. $\vec{u}=\vec{a}-\vec{b}$ *and* $\vec{v}=\vec{a}+\vec{b}$ *and* $|\vec{a}|=|\vec{b}|=2$, then $|(\vec{u}\times\vec{v})|$ is equal to (a)* $2\sqrt{16-\left(\vec{a}\cdot\vec{b}\right)^2}$ (b) $2\sqrt{4-\left(\vec{a}\cdot\vec{b}\right)^2}$ (c) $\sqrt{16-\left(\vec{a}\cdot\vec{b}\right)^2}$ (d) $\sqrt{4-\left(\vec{a}\cdot\vec{b}\right)^2}$

28. If $\vec{a}+\vec{b}+\vec{c}=0,\ and\ |\vec{a}|=2, |\vec{b}|=3, |\vec{c}|=5$ then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=$

(a) 19 (b)* -19 (c) 61 (d) -61

29. In ΔABC, $\vec{AB}=r\hat{i}+\hat{j},\ \vec{AC}=s\hat{i}-\hat{j}$. If the area of triangle is of unit magnitude, then

(a) $|r-s|=2$ (b) $|r+s|=1$ (c)* $|r+s|=2$ (d) $|r-s|=1$

30. $\hat{a},\hat{b},\hat{c}$ are three unit vectors, $\hat{a}\ and\ \hat{b}$ are perpendicular to each other and vector $\hat{c}$ is equally inclined to both $\hat{a}\ and\ \hat{b}$ at an angle θ. If $\hat{c}=\alpha\hat{a}+\beta\hat{b}+\gamma\left(\hat{a}\times\hat{b}\right)$, where α, β, γ are constants then

(a) $\alpha=\beta=-\cos\theta,\ \gamma^2=\cos 2\theta$ (b) $\alpha=\beta=\cos\theta,\ \gamma^2=\cos 2\theta$

(c)* $\alpha=\beta=\cos\theta,\ \gamma^2=-\cos 2\theta$ (d) $\alpha=\beta=-\cos\theta,\ \gamma^2=-\cos 2\theta$

31. If $\vec{a}$ makes an acute angle with $\vec{b}$ and $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}\ and\ \vec{r}\cdot\vec{a}=0$, then $\vec{r}$ is equal to

(a) $\vec{a}\times\vec{c}-\vec{b}$ (b) $\vec{c}\times\vec{a}$ (c)* $\vec{c}-\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\vec{b}$ (d) $\vec{c}+\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\vec{b}$

32. If $\vec{a}+2\vec{b}+3\vec{c}=0\ and\ \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to

(a) $3(\vec{a}\times\vec{c})$ (b) $6\left(\vec{c}\times\vec{b}\right)$ (c)* $2\left(\vec{a}\times\vec{b}\right)$ (d) $2\left(\vec{b}\times\vec{a}\right)$

33. Three consecutive vertices of a rhombus have the position vectors $\hat{i}+2\hat{j}+3\hat{k},\ 2\hat{i}+4\hat{j}+5\hat{k}$ and $4\hat{i}+5\hat{j}+3\hat{k}$, then the position vector of fourth vertex is

(a) $3\hat{i}+3\hat{j}+3\hat{k}$ (b) $3\hat{i}-3\hat{j}+3\hat{k}$ (c) $-3\hat{i}+3\hat{j}+3\hat{k}$ (d)* $3\hat{i}+3\hat{j}+\hat{k}$

34. If $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d},\ \vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$ and $\vec{a},\vec{b},\vec{c}$ are non-coplanar, then the sum $\vec{a}+b+\vec{c}+\vec{d}$ equals

(a)* $\vec{0}$ (b) $(\beta-1)\vec{d}+(\alpha-1)\vec{a}$ (c) $(\alpha-1)\vec{d}+(\beta-1)\vec{a}$

(d) $(\alpha-1)\vec{d}-(\beta-1)\vec{a}$

35. The volume of a tetrahedron, the position vectors of whose vertices are $5\hat{i}-\hat{j}+\hat{k},\ 7\hat{i}-4\hat{j}+7\hat{k},\ \hat{i}-6\hat{j}+10\hat{k}$ and $-\hat{i}-3\hat{j}+7\hat{k}$, is

(a) 15 (b) 3 (c)* 11 (d) 7

36. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be three non-zero vectors, no two of which are collinear and the vectors $\vec{a}+\vec{b}$ is collinear with $\vec{c}$ *while* $\vec{b}+\vec{c}$ is collinear with $\vec{a}$. Then $\vec{a}+\vec{b}+\vec{c}=$

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d)* none of these

37. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ *and* $m=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, then

(a)* m < 0 (b) m > 0 (c) m = 0 (d) m = 1

38. The vector $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}, \vec{b}=\hat{j}$ *and* $\vec{c}$ are such that $\vec{a}, \vec{b}, \vec{c}$ from a right handed system, then vector $\vec{c}$ is

(a) 0 (b) $y\hat{j}$ (c)* $z\hat{i}-x\hat{k}$ (d) $-z\hat{i}+x\hat{k}$

39. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ be vectors with magnitudes 3, 4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(a) 47 (b) 25 (c) 50 (d)* -25

40. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ *and* $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ *and* $\vec{c}$ whose projection on $\vec{a}$ is magnitude $\sqrt{2/3}$ is

(a)* $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$ (c) $-2\hat{i}+5\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

41. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ *and* $\vec{c}$ is

(a)* 3π/4 (b) π/4 (c) π/2 (d) π

42. A unit vector in xy-plane that makes an angle of $45°$ with the vector $\hat{i}+\hat{j}$ and an angle of $60°$ with the vector $3\hat{i}-4\hat{j}$ is

(a) $\hat{i}$ (b) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (c) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ (d)* none of these

43. If $\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}\times(\vec{C}\times\vec{A})$ and $[\vec{A}\,\vec{B}\,\vec{C}]\neq 0$ then $\vec{A}\times(\vec{B}\times\vec{C})$ is equal to

(a)* $\vec{0}$ (b) $\vec{A}\times\vec{B}$ (c) $\vec{B}\times\vec{C}$ (d) $\vec{C}\times\vec{A}$

44. If $\vec{u}, \vec{v}, \vec{w}$ are three non coplanar vectors , then $(\vec{u}+\vec{v}-\vec{w}).((\vec{u}-\vec{v})\times(\vec{v}-\vec{w}))$ equals (a)* $\vec{u}.(\vec{v}\times\vec{w})$ (b) $\vec{u}.\vec{v}\times\vec{w}$ (c) $3\vec{u}.\vec{v}\times\vec{w}$ (d) 0

45. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}$ be a coplanar unit vector perpendicular to $\vec{a}$ then $\vec{c}=$

(a)* $\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$ (b) $\frac{-\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$ (c) $\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ (d) $\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$

46. If $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}, \vec{c}=\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{n}$ be a unit vector such that $\vec{b}.\vec{n}=0, \vec{a}.\vec{n}=0$, then The the value of $|\vec{c}.\vec{n}|$ is

(a) 1 (b) 3 (c)* 5 (d) 2

47. If $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=4$ and $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to :

(a) 0 (b)* -25 (c) 25 (d) none of these

48. The magnitude of cross product of two vectors is $\sqrt{3}$ times the dot product. The angle between the vectors is:

(a) $\frac{\pi}{6}$ (b)* $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

49. The summation of two unit vectors is a third unit vector, then the modulus of the difference of the unit vectors is

(a)* $\sqrt{3}$ (b) $1-\sqrt{3}$ (c) $1+\sqrt{3}$ (d) $-\sqrt{3}\ \frac{2\hat{i}-3\hat{j}-6\hat{k}}{7}$

50. If M denotes the mid point of the line joining $A(4\hat{i}+5\hat{j}-10\hat{k})$ *and* $B(-\hat{i}+2\hat{j}+\hat{k})$, then equation of the plane through M and perpendicular to AB is

(a)* $\vec{r}\cdot(-5\hat{i}-3\hat{j}+11\hat{k})+\frac{135}{2}=0$ (b) $\vec{r}\cdot\left(\frac{3}{2}\hat{i}+\frac{7}{2}\hat{j}-\frac{9}{2}\hat{k}\right)+\frac{135}{2}=0$

(c) $\vec{r}\cdot(4\hat{i}+5\hat{j}-10\hat{k})+4=0$ (d) $\vec{r}\cdot(-\hat{i}+2\hat{j}+\hat{k})+4=0$