MATHEMATICS XII

1

Topic

Revision of Derivatives

Presented By

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Continuity

Def. In simple words , a function is continuous at a fixed point if we can draw graph of the function at around that point without lifting the pen from the plane of the paper.

Another Def. : A function is continuous at x=a if the function is defined at x=a and if the value of the function at x=a is equal the limit of the function at x=a.

Discontinuity

Note 1 : If f is discontinuous at x=a then a is called point of discontinuity.

Note 2 : A function f is discontinuous at x=a in following cases :

(i) f is not defined at x=a i.e. f(a) does not exist.

(ii) Limit of f(x) at x=a does not exit.

(iii) Limit of f(x) at x=a exits but not equal to f(a).

Cases of Discontinuity

Limit of f(x) at x=a does not exit.

This happens in following cases :

Case I $\underset{x \to a^+}{Lt} f(x)\, does\ not\ exit$

Case II $\underset{x \to a^-}{Lt} f(x)\, does\ not\ exit$

Case III $\underset{x \to a^+}{Lt} f(x)\ and\ \underset{x \to a^-}{Lt} f(x)\ both\ exit\ and\ are\ not\ equal$

Kinds of Discontinuity

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1. Removable Discontinuity : Some times a function f is not defined at x=a or f(a) is defined in such a way that it is not equal to limit of f(x) at x=a, then this discontinuity can be removed by defining f(a) in such a way that it may equal to limit of f(x) at x=a.

2. Non Removable Discontinuity : This is of two kinds. (i) Discontinuity of first kind

(ii) Discontinuity of second kind

Non Removable Discontinuity

(i) Discontinuity of first kind :

$$\mathop{Lt}_{x \to a^+} f(x) \text{ and } \mathop{Lt}_{x \to a^-} f(x) \text{ both exit and are not equal}$$

(ii) Discontinuity of second kind :

$$\mathop{Lt}_{x \to a^+} f(x) \text{ does not exit}$$

or

$$\mathop{Lt}_{x \to a^-} f(x) \text{ does not exit}$$

Continuity in interval

(i) Open interval : A real valued function f defined on open interval (a,b)is said to be continuous in on (a,b) if it is continuous at $x=c$ for all $c \in (a,b)$

(ii) Closed interval : A real valued function f defined on closed interval [a,b]is said to be continuous in on [a,b] if (i) f is right continuous at $x=a$.

(ii) f is left continuous at $x=b$.

(iii) f is continuous at $x=c$ for all $c \in (a,b)$

Such have continuous graph on [a,b]

Algebra of Continuity

If f , g are two continuous functions at x=a

then

(i) kf is continuous function at x=a ,k $\in$ R.

(ii) (f $\pm$g) is continuous function at x=a .

(iii) fg is continuous function at x=a .

(iv) f/g is continuous function at x=a , g(a) $\neq$

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Prove that
A constant function is continuous everywhere.

Let f(x) =c be a constant function $x \in R$.

Let a be any real number.

Now $\lim_{x \to a} f(x) = \lim_{x \to a} (c) = c$

Also f(a) =c.

$$\lim_{x \to a} f(x) = f(a) \ \forall a \in R$$

$\Rightarrow$ f is continuous everywhere.

Is the function $f(x)=x^2-\sin x+5$ continuous at $x=\pi$

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We have $f(x)=x^2-\sin x+5$.

$$\lim_{x\to\pi} f(x) = \lim_{x\to\pi}(x^2 - \sin x + 5)$$

$$= \pi^2-\sin \pi+5$$

$$= \pi^2-0+5 \quad = \pi^2+5$$

Also $f(\pi)=\pi^2-\sin \pi+5 = \pi^2-0+5 \quad = \pi^2+5$

$\therefore \lim_{x\to\pi} f(x) = f(\pi) \qquad \Rightarrow$ f is continuous at π

Board Questions

Q. 1 If f(x) is continuous at x=0,

find the value of k. (Mar 2008)

$$\text{Given: } f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2} & , x \neq 0 \\ k & , x = 0 \end{cases}$$

Board Questions

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Q. 1 (Mar 2008)

$$\text{Given: } f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2} & , x \neq 0 \\ k & , x = 0 \end{cases}$$

If f(x) is continuous at x=0, find the value of k.

Sol. $$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1-\cos 2x}{x^2} = \lim_{x \to 0} \frac{2\sin^2 x}{x^2}$$

$$= \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$

Board Questions

Sol. $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1 - \cos 2x}{x^2} = \lim_{x \to 0} \frac{2\sin^2 x}{x^2}$

$$= \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$

Now f(0) = k. (given)

Because function is continuous at x=0

So Limiting value of f(x) is same as f(0).

$$\lim_{x \to 0} f(x) = f(0)$$

So k=2

Ans.

Board Questions

Q. 1 Discuss continuity of f(x) at x=0 (Mar 2007)

$$f(x) = \begin{cases} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sin x} & , x \neq 0 \\ 1 & , x = 0 \end{cases}$$

Board Questions

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Q. 1 Discuss continuity of f(x) at x=0 (Mar 2007)

$$f(x) = \begin{cases} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sin x} & ,x\neq 0 \\ 1 & ,x=0 \end{cases}$$

Sol. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sin x}$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sin x} \text{ x } \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

Board Questions

Sol. $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sin x}$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{\sin x} \times \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\lim_{x\to 0} \frac{1+x-(1-x)}{\sin x\ (\sqrt{1+x}+\sqrt{1-x})}$$

$$\lim_{x\to 0} \frac{2x}{\sin x\ (\sqrt{1+x}+\sqrt{1-x})}$$

Board Questions

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Sol. $\lim\limits_{x\to 0} \dfrac{1 + x - (1 - x)}{\sin x\ (\sqrt{1 + x} + \sqrt{1 - x})}$

$\lim\limits_{x\to 0} \dfrac{2x}{\sin x\ (\sqrt{1 + x} + \sqrt{1 - x})}$

$= \lim\limits_{x\to 0} \dfrac{x}{\sin x} \cdot \dfrac{2}{\sqrt{1+x} + \sqrt{1-x}}$

$= 1 \cdot \dfrac{2}{\sqrt{1 + 0} + \sqrt{1 - 0}} = 1$

Board Questions

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Sol. $= 1 \cdot \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = 1$

$$= \lim_{x \to 0} f(x) = 1$$

Now $f(0) = 1$

$$\therefore \lim_{x \to 0} f(x) = f(0)$$

$\therefore$ f(x) is continuous at $x = 0$

Board Questions

Discuss the continuity of f(x) at x=0 , where

$$f(x) = \begin{cases} \dfrac{2|x| + x^2}{x} & , \text{if } x \neq 0 \\ 0 & , \text{if } x = 0 \end{cases}$$

(Mar 2003)

Solution :- Here function contain modulus of x hence we have find its limit from both sides because I x I=x if x≥0 and I x I =-x if x<0 .

Continued

Sol. $\text{L. H. L.} = \lim_{x \to 0^-} f(x)$

$$= \lim_{x \to 0^-} \frac{2|x| + x^2}{x}$$

$$= \lim_{x \to 0^-} \frac{2(-x) + x^2}{x} \qquad |x| = -x \text{, if } x \le 0$$

$$= \lim_{x \to 0^-} \frac{2x(-1 + x)}{x} = \lim_{x \to 0^-} 2(-1 + x)$$

$$= 2(-1 + 0) = -2$$

Continued

Right limit

$$\text{R. H. L.} = \lim_{x\to 0^+} f(x)$$

$$= \lim_{x\to 0^+} \frac{2|x| + x^2}{x}$$

$$= \lim_{x\to 0^+} \frac{2x + x^2}{x} \qquad |x| = x \text{, if } x \geq 0$$

$$= \lim_{x\to 0^+} \frac{2x(1 + x)}{x} = \lim_{x\to 0^+} 2(1 + x)$$

$$= 2(1 + 0) = 2$$

Continued

So L.H.L. $\neq$ R.H.L.

Limit at x=0 does not exist.

Hence from def. of continuity f(x) is discontinuous at x=0.

Here we need not to find f(0).

Question

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Q.No. 1 Find K so that function is continuous at point $x=\pi/2$

$$f(x)=\begin{cases} \dfrac{K\cos x}{\Pi-2x} & \text{if } x\neq\pi/2 \\ 3 & \text{if } x=\pi/2 \end{cases}$$

Q. No.2 Examine the continuity at $x=0$

$$f(x)=\begin{cases} \text{Sin } x-\cos x & \text{if } x\neq 0 \\ -1 & \text{if } x=0 \end{cases}$$

Question

Q.No. 1

Find the all points of discontinuity of function

$$f(x) = \begin{cases} 2x+3 & \text{if } x \leq 0 \\ 2x-3 & \text{if } x > 0 \end{cases}$$

Q. No.2Find the value of a and b if function is continuous.

$$f(x)= \begin{cases} 5 & \text{If } x \leq 2 \\ ax + b & \text{if } 2< x <10 \\ 21 & \text{If } x\geq 10 \end{cases}$$

Differentiation
Introduction

- Increment
- Differential Co-efficient
- Notation

Definition

- Let f(x) be function defined then

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

If it exit is called differential coefficient of y w.r.t x and is denoted as f'(x).

Steps for derivative

- 1. Put y = given function.
- Change x to Δx and y to Δy
- Subtract (1) from (2) and obtain Δy and simplify.
- Divide both sides by Δx.
- Take limits both sides as $\Delta x \to 0$ keeping in mind

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

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Physical Meaning

- Imagine x is time and f(x) is distance travelled in time x.
- Let $f(x+\Delta x)$ be distance in $x+\Delta x$ time.
- Then distance $f(x+\Delta x) - f(x)$ is travelled in time Δx.
- Speed =Distance travelled / time.
- Speed in interval=$(f(x+\Delta x) - f(x)) / \Delta x$
- Speed at a point = $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

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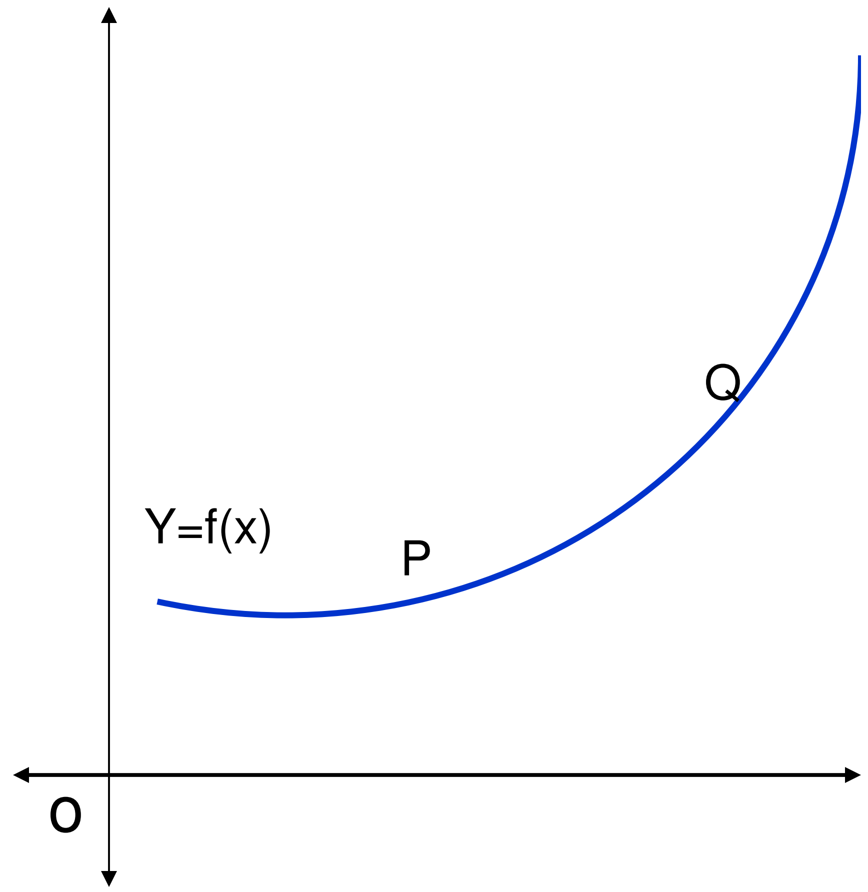
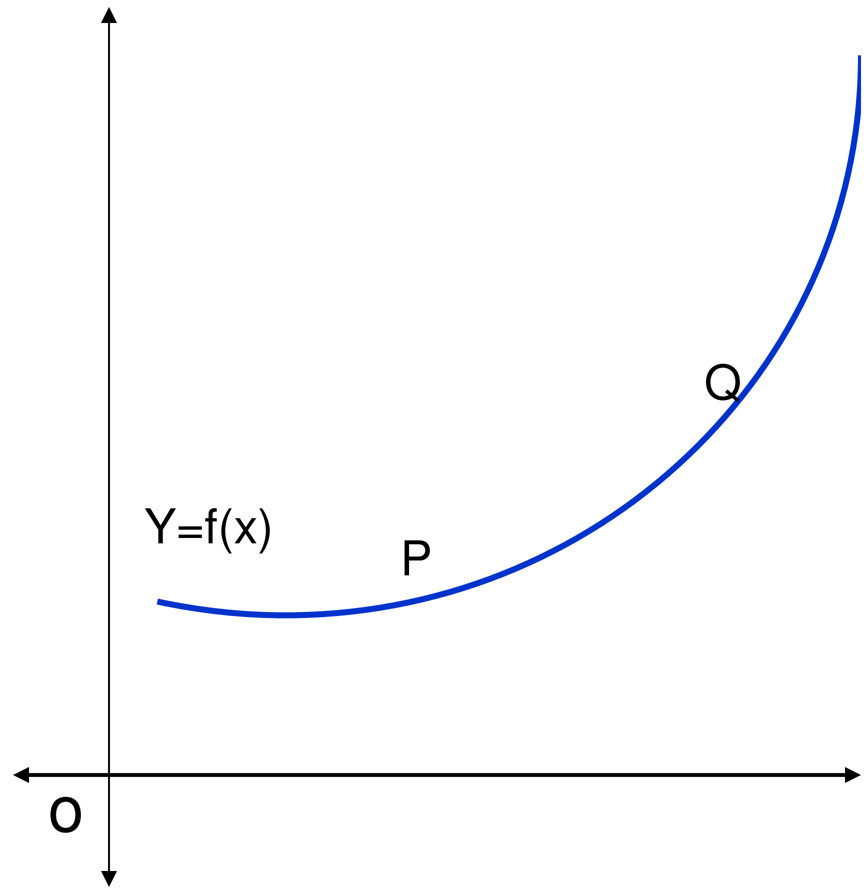
Geometrical significance

- Let y=f(x) be function whose graph in xy plane is shown by curve PQ



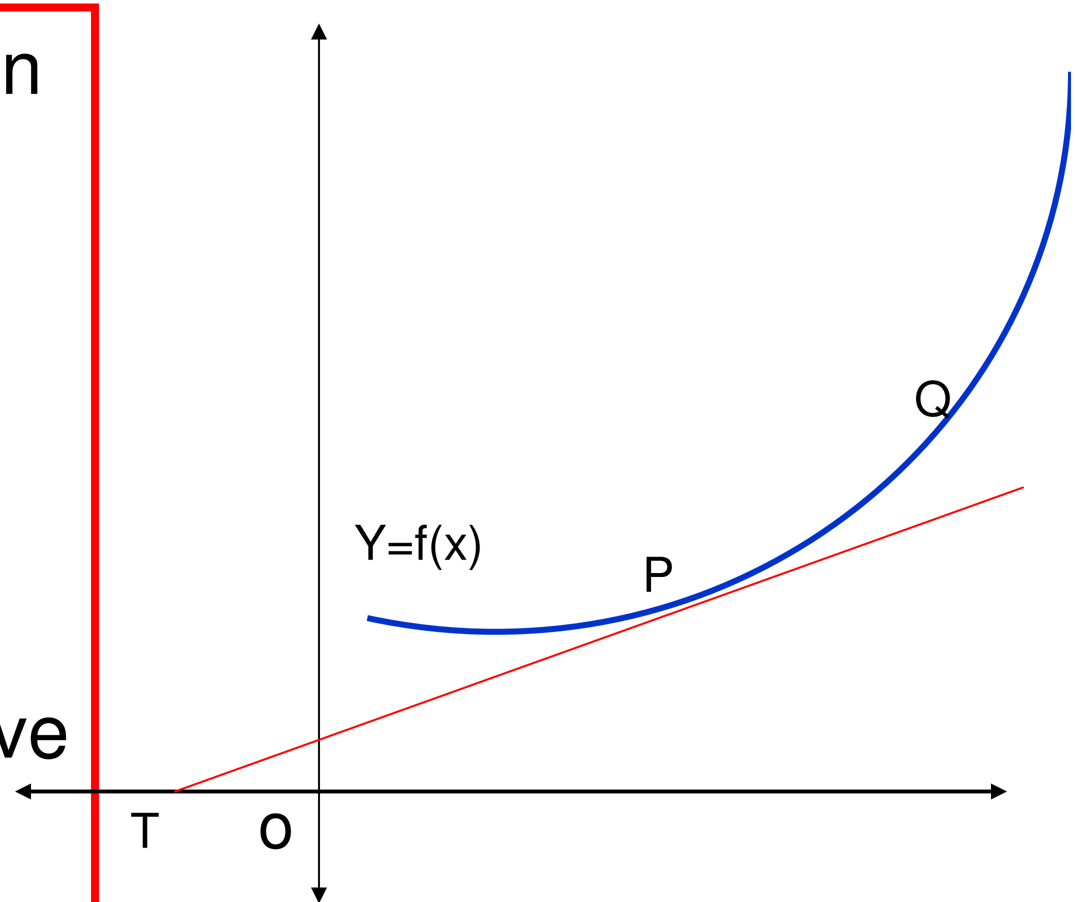
Geometrical significance

- Let y=f(x) be function whose graph in xy plane is shown by curve
- Let P(c,f(c)) and Q(c+h, f(c+h)) be ponits on curve



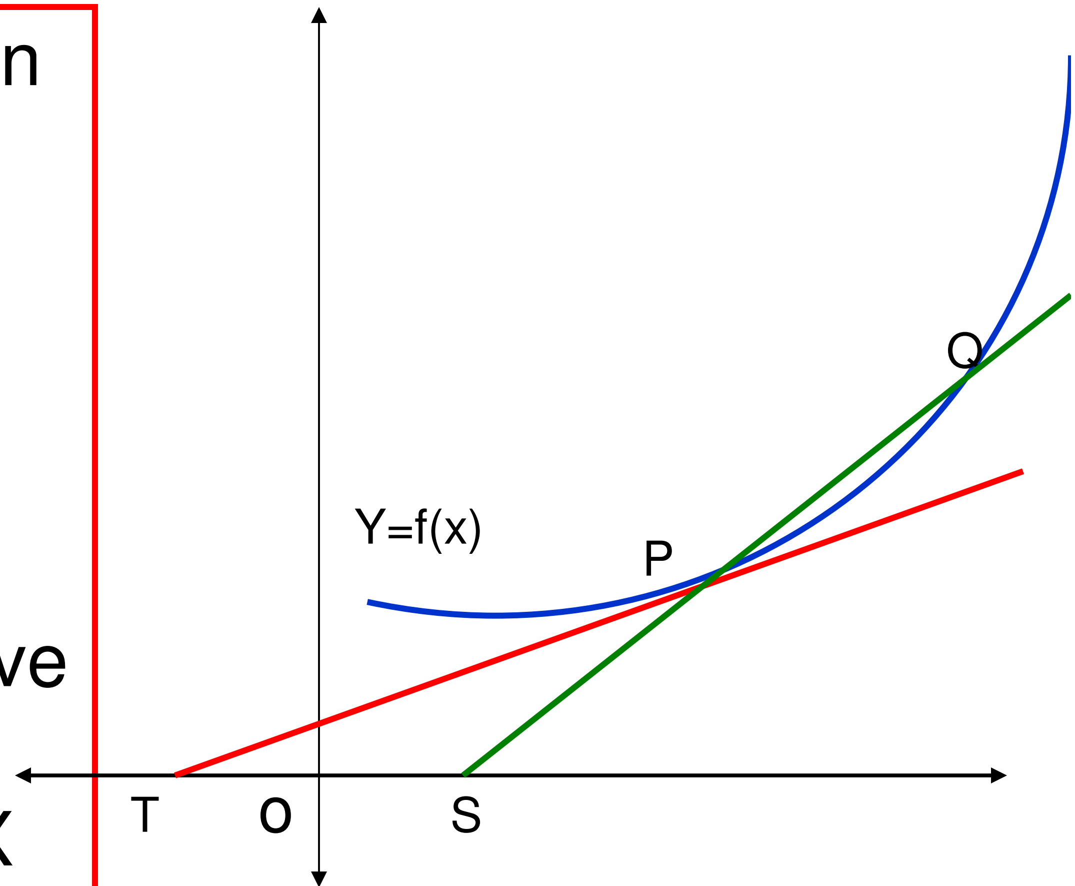
Geometrical significance

- Let y=f(x) be function whose graph in xy plane is shown by curve
- Let P(c,f(c)) and Q(c+h, f(c+h)) be ponits on curve
- PT is tangent to curve at P



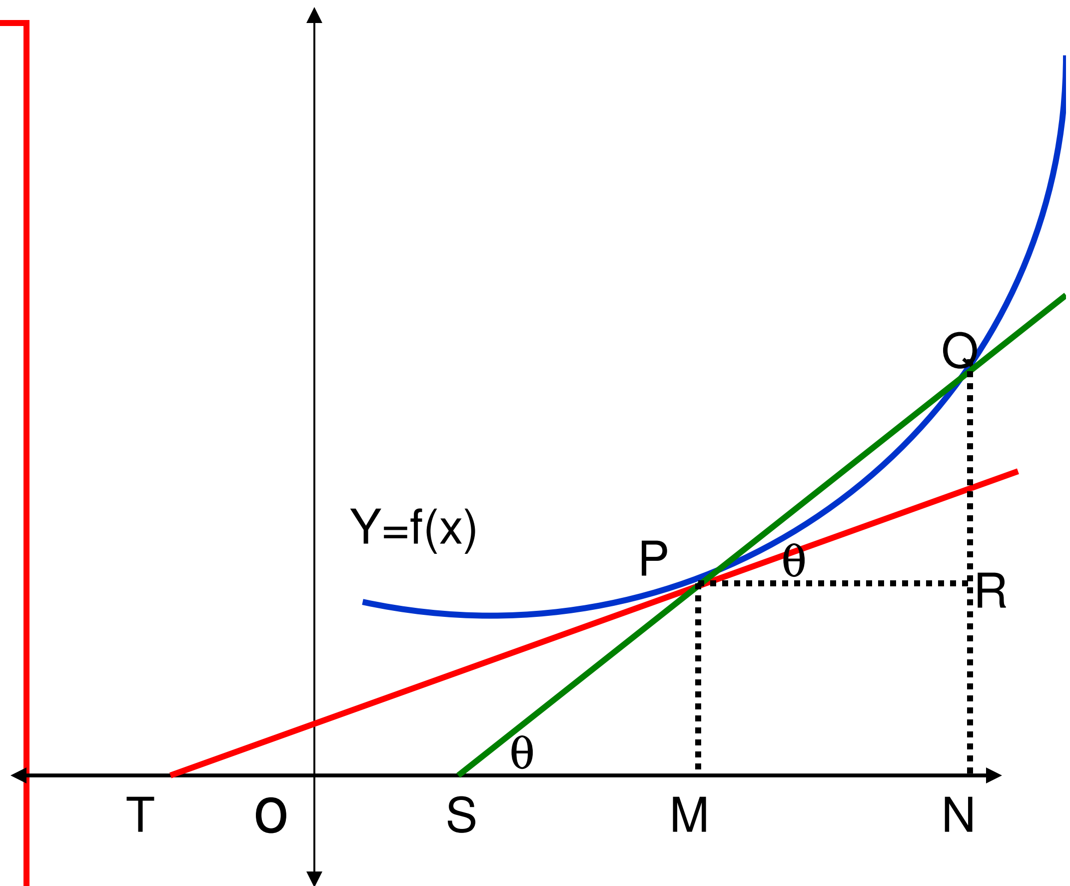
Geometrical significance

- Let y=f(x) be function whose graph in xy plane is shown by curve
- Let P(c,f(c)) and Q(c+h, f(c+h)) be ponits on curve
- PT is tangent to curve at P
- PQ chord meet OX at S



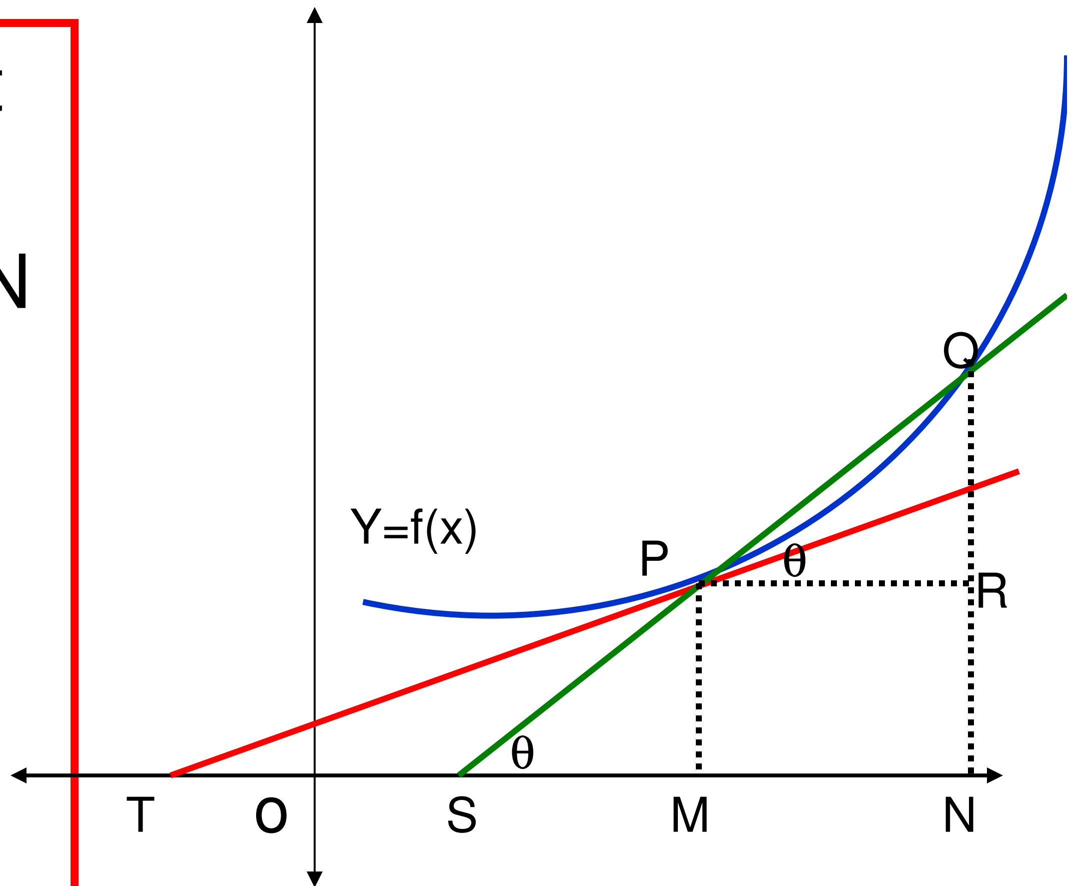
Geometrical significance

- PQ chord meet OX at S
- Draw PM⊥OX, QN ⊥OX, PR ⊥QN.
- ∠XSQ= ∠ RPQ=θ
- ∠XTP=α
- PR=MN=ON-OM =c+h-c = h
- RQ=QN-RN=QN-MP = f(c+h)-f(c)



Geometrical significance

- PQ chord meet OX at S
- Draw PM⊥OX, QN ⊥OX, PR ⊥QN.
- $\angle XSQ = \angle RPQ = \theta$
- $\angle XTP = \alpha$
- PR=MN=ON-OM =c+h-c = h
- RQ=QN-RN=QN-MP = f(c+h)-f(c)

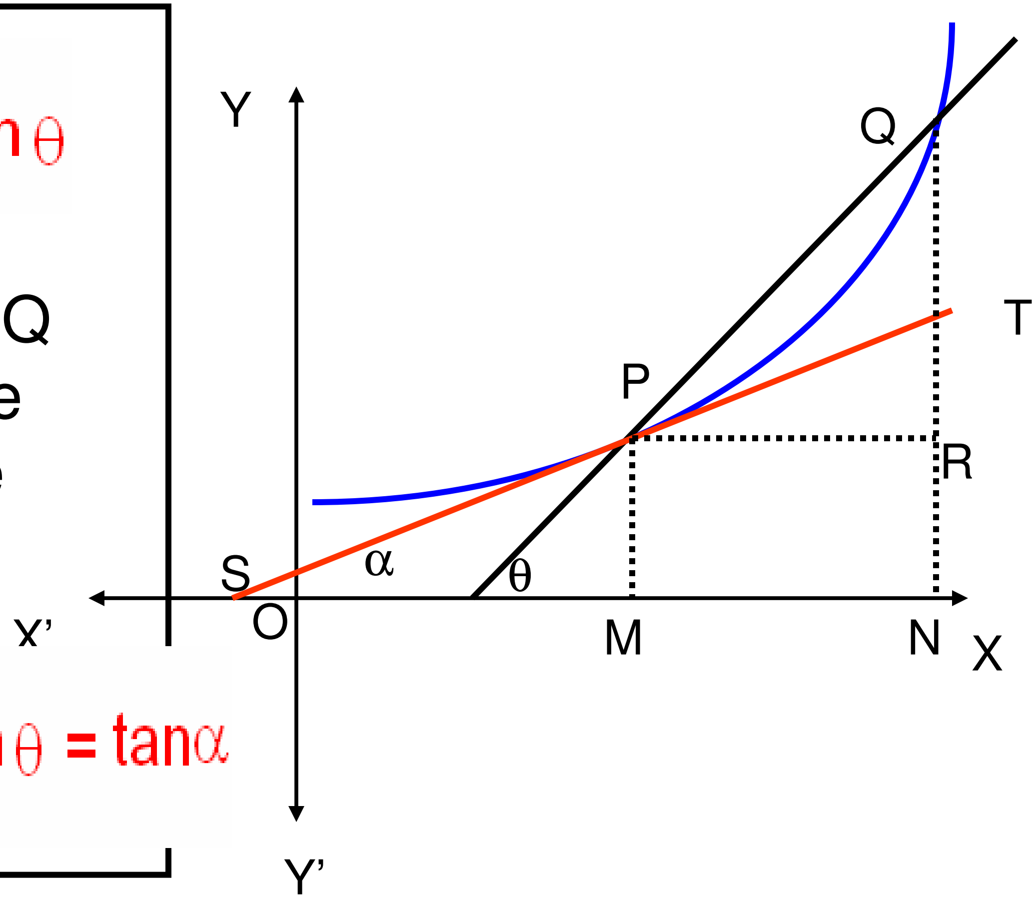


Geometrical significance

$$\frac{QR}{PR} = \frac{f(c+h) - f(c)}{h} = \tan\theta$$

•Now if Q approaches P along the curve the line PQ become the tangent to the curve at P in limiting case $\theta \rightarrow \alpha$ as $h \rightarrow 0$

$$\underset{h\rightarrow 0}{Lt} \frac{f(c+h) - f(c)}{h} = \underset{\theta\rightarrow\alpha}{Lt} \tan\theta = \tan\alpha$$



Left hand derivative at a point

- A function f is said to be derivable to the left of a point $c \in D_f$ iff

$$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$$

exists finitely and is denoted by Lf'(c) and is called left hand derivative of f w.r.t. x at x=c

Right hand derivative at a point

- A function f is said to be derivable to the right of a point $c \in D_f$ iff

$$\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c} = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$$

exists finitely and is denoted by Rf'(c) and is called right hand derivative of f w.r.t. x at x=c .

Derivative at a point

- A function f is said to be derivable at a point $c \in D_f$ iff

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

exists finitely and is denoted by f'(c) and is called derivative of f w.r.t. x at x=c

Derivative and Continuity

- A function which is derivable at a point is a continuous at that point. But its converse may or may not be true.

Some Typical examples

- 1 . Show that the function defined by

  $f(x) = 3-2x$ if $x<2$

  $3x-7$ if $x \geq 2$

  is continuous at $x=2$ but not derivable at $x=2$.

Another example

- Find left and right derivatives of

$$f(x) = \begin{cases} 2x+1 & \text{if } x<1 \\ 6x+7 & \text{if } x\geq 1 \end{cases}$$

at $x=1$. Is f is derivable at $x=1$?

Different names of first principle

1. By First Principle
2. From Definition
3. By Delta Method
4. By Ab-inito

Derivative by First Principle

1. Put y= given function.
2. Change x to Δx and y to Δy
3. Subtract (1) from (2) and obtain Δy and simplify.
4. Divide both sides by Δx.
5. Take limits both sides as $\Delta x \to 0$ keeping in mind

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

1 $Y=f(x)$

2 $Y+ \Delta y = f(x+ \Delta x)$

3 $Y+ \Delta y - y = f(x+ \Delta x) - f(x)$

4 $$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

5 $$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

6 $$\frac{dy}{dx} = f'(x)$$

Some Standard Results

- $$\frac{d}{dx}(x)^n = n(x)^{n-1}$$

$$\frac{d}{dx}a^x = a^x \cdot \log a$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\log_a x = \frac{1}{x}\log_a e$$

$$\frac{d}{dx}\log_e x = \frac{1}{x}\log_e e$$

$$\frac{d}{dx}\log x = \frac{1}{x}$$

Some Standard Results

$$\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1}.a$$

$$\frac{d}{dx}a^{ax+b} = a^{ax+b}.\log a.a$$

$$\frac{d}{dx}e^{ax+b} = e^{ax+b}.a$$

$$\frac{d}{dx}\log_a(ax+b) = \frac{1}{ax+b}.a.\log_a e$$

$$\frac{d}{dx}\log(ax+b) = \frac{1}{ax+b}.a$$

Derivative of Trigonometric Functions

Derivative

1 $D(\sin x) = \cos x$

2 $D(\cos x) = -\sin x$

$$D = \frac{d}{dx}$$

3 $D(\tan x) = \sec^2 x$

4 $D(\cot x) = -\operatorname{cosec}^2 x$

5 $D(\sec x) = \sec x \,.\, \tan x$

6 $D(\operatorname{cosec} X) = -\operatorname{cosec} x \,.\, \cot x$

Derivative of Trigonometric Functions

1 $D(\sin ax+b) = \cos(ax+b).a$

2 $D(\cos ax+b) = -\sin(ax+b)$ $D = \frac{d}{dx}$

3 $D(\tan ax+b) = \sec^2(ax+b).a$

4 $D(\cot ax+b) = -\text{cosec}^2(ax+b).a$

5 $D(\sec ax+b) = \sec(ax+b).\tan(ax+b).a$

6 $D(\text{cosec } ax+b) = -\text{cosec}(ax+b).\cot(ax+b).a$

Derivative of Composite Functions

Let $f(x) = (goh)(x)$

$= g(h(x))$

Then $f'(x) = g'(h(x)) \cdot h'(x)$

Chain Rule

Let $y = f(t)$

And $t = g(x)$ be two functions.

We want to find derivative of y w.r.t. x

$dy/dt = f'(t)$

$dt/dx = g'(x)$ then

$dy/dx = dy/dt * dt/dx$

$= f'(t) * g'(x)$

$= f'(g(x)) * g'(x)$

Generalised Chain Rule

Let y = f(t),

t = g(u),

u =h(x),

And we want derivative of y w.r.t. x.Then

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{du} \times \frac{du}{dx}$$

.

Questions on chain rule

Q No. 1.

$$\text{Find } \frac{dy}{dx} \text{ when } y = \frac{v^4}{4} \text{ and } v = \frac{2}{3}x^2 + 5$$

Q No. 2 Differentiate by chain rule

$$y = \sqrt{15\,x^2 - x + 1}$$

Questions on Derivative

Q No. 1

Differentiate $\log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x

Q No. 2

Differentiate : $e^{3x} \log(\sin 2x)$

Important Results of Derivatives

- $D(\text{Constant}) = 0$
- $D(u \pm v) = D(u) \pm D(v)$
- $D(u \times v) = u \times D(v) + v \times D(u)$

$$D\frac{u}{v} = \frac{v \times D(u) - u \times D(v)}{v^2}$$

$$D = \frac{d}{dx}$$

Derivative of parametric equations

- Let $x = f(t)$ and $y = g(t)$ be two functions of t
- Then $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$$

Questions on Parametric function

Q No. 1.

$$\text{Find } \frac{dy}{dx}, \textit{when } x = a\cos\theta \textit{ and } y = b\sin\theta$$

Q. No. 2

$$\text{If } x = \frac{1+\log t}{t^2}, y = \frac{3+2\log t}{t},$$

$$t > 0, \text{ prove that } y\frac{dy}{dx} - 2x\left(\frac{dy}{dx}\right)^2$$

=1

Derivative of one function w.r.t. other function

- Put one function of x is equal to y and put other function of x is equal to u
- Then $$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}}$$

Example of Derivative of a function w.r.t. function

- Differentiate $7x^5-11x^2$ w.r.t. $7x^2 - 15x$
- Let $u = 7x^5-11x^2$, $v = 7x^2 - 15x$

$$\frac{du}{dx} = 35x^4 - 22x$$

$$\frac{dv}{dx} = 14x - 15$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{35x^4 - 22x}{14x - 15}$$

Derivative w.r.t. another function

Q No. 1.

$$Differentiate\ tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) w.r.t.$$

$$sin^{-1}\frac{2x}{1+x^2}$$

Q No. 2

Differentiate log(xe^x) w.r.t. x log x

Derivative of Implicit Functions

- Explicit functions -- When a relationship between x and y is expressed in a way that it is easy to solve for y and write y = f(x)
- Implicit functions - When a relationship between x and y is expressed in a way that it is not easy to solve for y and y is not expressed in terms of x.

Examples of Implicit functions

$$x^6 + y^6 + 6x^2y^2 = 16$$

$$e^x + e^y = e^{x+y}$$

Logarithmic Derivative

Q No. 1

Differentiate : $y = x^x$

Q No. 2

Differentiate : $y = (x^x)^x$

Logarithmic Derivative

Q No. 1

Differentiate : $y = x^{\log x} + (\log x)^x$

Q No. 2

Differentiate : $y = \sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$

Derivative of Inverse T Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$ Where $-1<x<1$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$ Where $-1<x<1$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$ Where $-\infty<x<\infty$

$$\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$$ Where $-\infty<x<\infty$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$ Where $x>1$ or $x<-1$

$$\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$$ Where $x>1$ or $x<-1$

Derivative of Inverse T Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$ Where $-1<x<1$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$$ Where $-1<x<1$

Derivative of Inverse T Functions

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \text{Where } -\infty < x < \infty$$

$$\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \qquad \text{Where } -\infty < x < \infty$$

Derivative of Inverse T Functions

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$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}} \quad \text{Where } x>1 \text{ or } x<-1$$

$$\frac{d}{dx}\left(\operatorname{cosec}^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}} \quad \text{Where } x>1 \text{ or } x<-1$$

Some Important substitutions

- In case of

(1) $a^2 + x^2$, put $x = a \tan\theta$

(2) $\sqrt{a^2 - x^2}$, put $x = a \sin\theta$

(3) $\sqrt{x^2 - a^2}$,put $x = a \sec\theta$

1 Properties of Inverse T-Functions

- $\sin(\sin^{-1}x) = x, \ x \in [-1,1]$
  and $\sin^{-1}(\sin x) = x, \ x \in [-\pi/2, \pi/2]$

  Same result is true for other five trigonometric ratios

2 Properties of Inverse T-Functions

- $\operatorname{cosec}^{-1}x = \sin^{-1}(1/x)$ $x \geq 1$ or $x \leq -1$
- $\cos^{-1} x = \sec^{-1}(1/x)$ $x \geq 1$ or $x \leq -1$
- $\cot^{-1}x = \tan^{-1}(1/x)$ $x > 0$

3 Properties of Inverse T-Functions

- $\sin^{-1}(-x) = -\sin^{-1}x, \quad x \in [-1,1]$
- $\cos^{-1}(-x) = \pi - \cos^{-1}x \quad x \in [-1,1]$
- $\tan^{-1}(-x) = -\tan^{-1}x \quad x \in R$
- $\cot^{-1}(-x) = -\cot^{-1}x \quad x \in R$
- $\sec^{-1}(-x) = \pi - \sec^{-1}x \quad |x| \geq 1$
- $\text{cosec}^{-1}(-x) = -\text{cesec}^{-1}x \quad |x| \geq 1$

4 Properties of Inverse T-Functions

- $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ •$x \in [-1,1]$
- $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ •$x \in R$
- $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$ $|x| \geq 1$

5 Properties of Inverse T-Functions

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, \qquad xy<1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}, \qquad xy>-1$$

$$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, \qquad |x|<1$$

6 Properties of Inverse T-Functions

$$2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}, \quad |x| < 1$$

$$2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}, \quad |x| < 1$$

7 Properties of Inverse T-Functions

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}]$$

$$\sin^{-1}x - \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$$

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{(1-x^2)(1-y^2)}]$$

$$\cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{(1-x^2)(1-y^2)}]$$

Derivative of Inverse T-Function

Q No. 1

Differentiate $y = \sin^{-1}\left(\frac{a + b\cos x}{b + a\cos x}\right)$

Q No. 2

Differentiate $y = \cos^{-1}\left(\frac{3\cos x - 4\sin x}{5}\right)$

Derivative of Inverse T-Function

Q No. 1

$$\text{Differentiate} \quad \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right) \quad \text{w.r.t. } x$$

Q No. 2

$$\text{Differentiate} \quad \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) \quad \text{w.r.t.} \quad \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Multiple Angle Formulae

1 $\text{Sin } 2A = 2 \text{ Sin } A \text{ Cos } A$

2 $\text{Cos } 2A = \text{Cos}^2 A - \text{Sin}^2 A$

$= 2 \text{ Cos}^2 A - 1$

$= 1 - 2 \text{ Sin}^2 A$

3 $\text{Tan } 2A = \dfrac{2 \text{ Tan } A}{1 - \text{Tan}^2 A}$

Multiple Angle Formulae

$$\text{Sin } 3A = 3 \text{ Sin } A - 4 \text{ Sin}^3 A$$

$$\text{Cos } 3A = 4 \text{ Cos}^3 A - 3 \text{ Cos } A$$

$$\text{Tan } 3A = \frac{3 \text{ Tan } A - \text{Tan}^3 A}{1 - 3 \text{ Tan}^2 A}$$

Multiple Angle Tan Form

$$\text{Sin } 2A = \frac{2 \text{ Tan } A}{1 + \text{Tan}^2 A}$$

$$\text{Cos } 2A = \frac{1 - \text{Tan}^2 A}{1 + \text{Tan}^2 A}$$

$$\text{Tan } 2A = \frac{2 \text{ Tan } A}{1 - \text{Tan}^2 A}$$

Another Form

$$\text{Sin A} = \pm\sqrt{\frac{1 - \text{Cos 2A}}{2}}$$

$$\text{Cos A} = \pm\sqrt{\frac{1 + \text{Cos 2A}}{2}}$$

$$\text{Tan A} = \pm\sqrt{\frac{1 - \text{Cos 2A}}{1 + \text{Cos 2A}}}$$

$\tan(\pi/4 \pm A)$

$$\tan(\pi/4 + A) = \frac{1 + \tan A}{1 - \tan A}$$

$$\tan(\pi/4 - A) = \frac{1 - \tan A}{1 + \tan A}$$

Derivative of T-functions

Q No. 1

Differentiate w.r.t. x : $y = \sin^3 x + \cos^6 x$

Q No. 2

Differentiate : $y = \log \sqrt{\dfrac{1+\sin x}{1-\sin x}}$

Questions

Q No. 1

Differentiate : $y = \dfrac{1}{\sqrt{x^2+1} + \sqrt{x^2+2}}$

Q No. 2

Differentiate : y

$$= x\sqrt{a - x^2} + (x+2)\sqrt{4-x}$$

Q No. 3

Differentiate : y

$$= \sqrt{(x-1)(x-2)(x-3)(x-4)}$$

Questions

Q No. 1 Differentiate : $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

(1) *Product rule* (2) *Expanding the product*

(3) *By Logarithmic differentiation*

Q No. 2 Differentiate : $y = \sin x^\circ$

Q No. 3 Differentiate :

$$x = \cos\theta + \cos 2\theta, \quad y = \sin\theta + \sin 2\theta$$

Questions

Q No. 1 Differentiate :

$$y = x^{\sin x} + (\sin x)^{\cos x}$$

Q No. 2 Differentiate :

$$y = \sin\left(2\sin^{-1}x\right)$$

Q No. 3

Differentiate : $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

w. r. t. $\tan^{-1}\dfrac{2x\sqrt{1-x^2}}{1-2x^2}$

Derivatives of Higher order

Q No. 1

If $y = e^{ax} \sin bx$ prove that

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0$$

Q No. 2

If $y = e^{m \sin^{-1} x}$ prove that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$

Derivatives of Higher order

Q No. 1

$$\text{If } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ show that } \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^2}$$

Q No. 2

$$\text{If } y = \sin(m \sin^{-1} x) \quad \text{prove that}$$

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$

Rolle's Theorem

Statement: If a function f(x) defined on [a,b] is such that

(i) f(x) is continuous in closed interval [a,b]

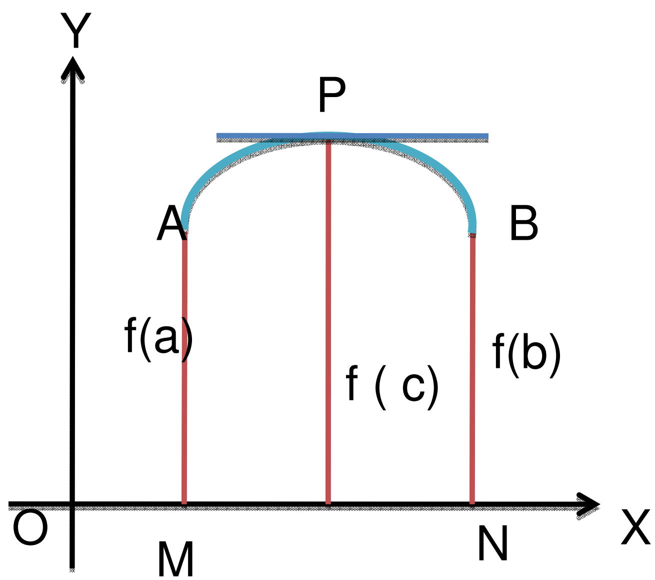
(ii) f(x) is derivable in open interval(a,b)

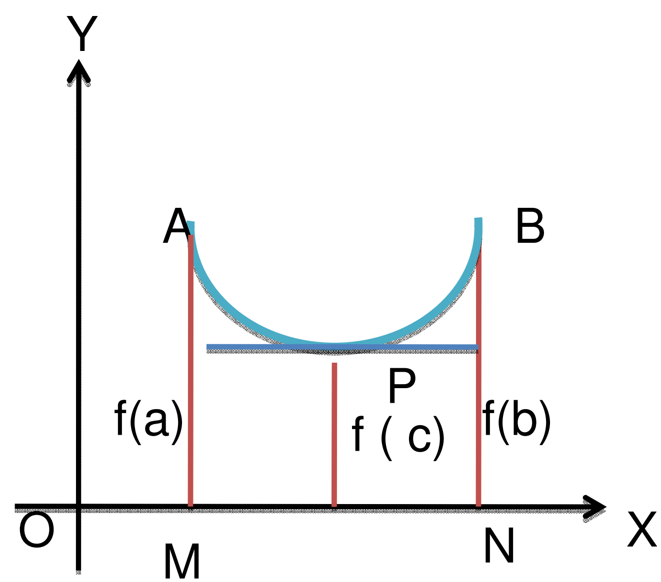
(iii) f(a)= f(b)

then there exists at least one real number c (a,b) such that f'(c)= 0

Rolle's Theorem Geometrical Interpretation

Let AB be the graph of function y=f(x) such that the point A and B of the graph correspond to the numbers a and b of the interval [a,b]



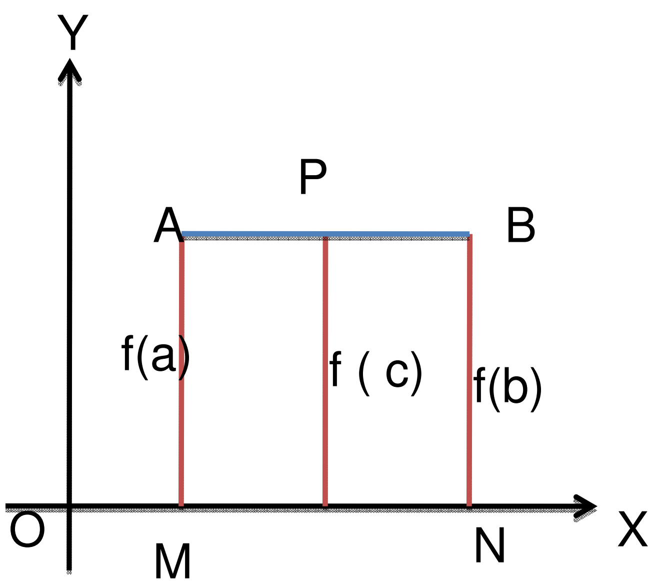


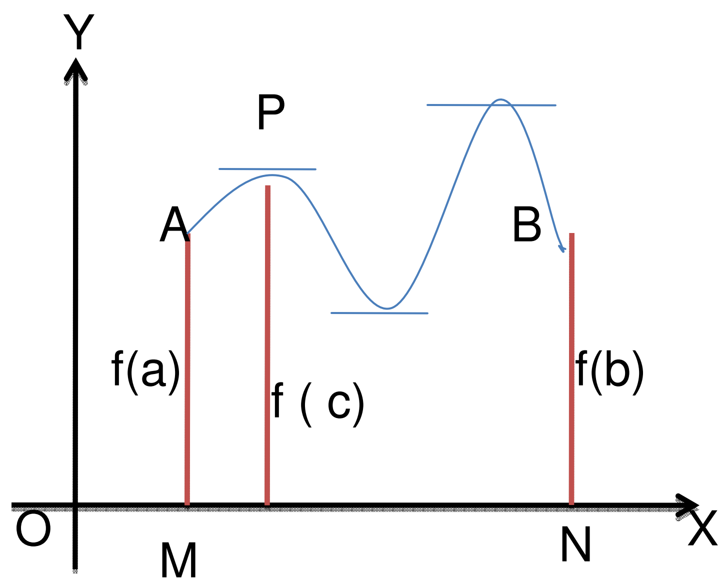
Rolle's Theorem Geometrical Interpretation

f(x) is continuous in the interval [a,b]

its graph is a continuous curve between A

and B.





Rolle's Theorem Geometrical Interpretation

Again as f(x) is derivable in open interval (a,b), therefore, graph of f(x) has a unique tangent at every point between A and B.

Because f(a)=f(b)

$\therefore$ AM=BN

From four figures it is clear that there is at least one point P on the curve between A and B , the tangent is parallel to x-axis.

$\therefore$ Slope of tangent at P=0

$\therefore$ f'(c)=0 ,where c is abscissa of P.

Varify Rolle's theorem in following cases

Q No. 1

$f(x)=x^3+3x^2-24x-80$ in interval [-4,5]

Q No. 2

$f(x)= \sin x-\sin 2x$ in $0 \le x \le 2\pi$

Q No. 3

Discuss the applicability of Rolle's theorem to the function

$$f(x) = \frac{x(x-2)}{x-1} \quad \text{on} \quad [0,2]$$

Lagrange's mean value Theorem

Statement: If a function f(x) defined on [a,b] is such that

(i) f(x) is continuous in closed interval [a,b]

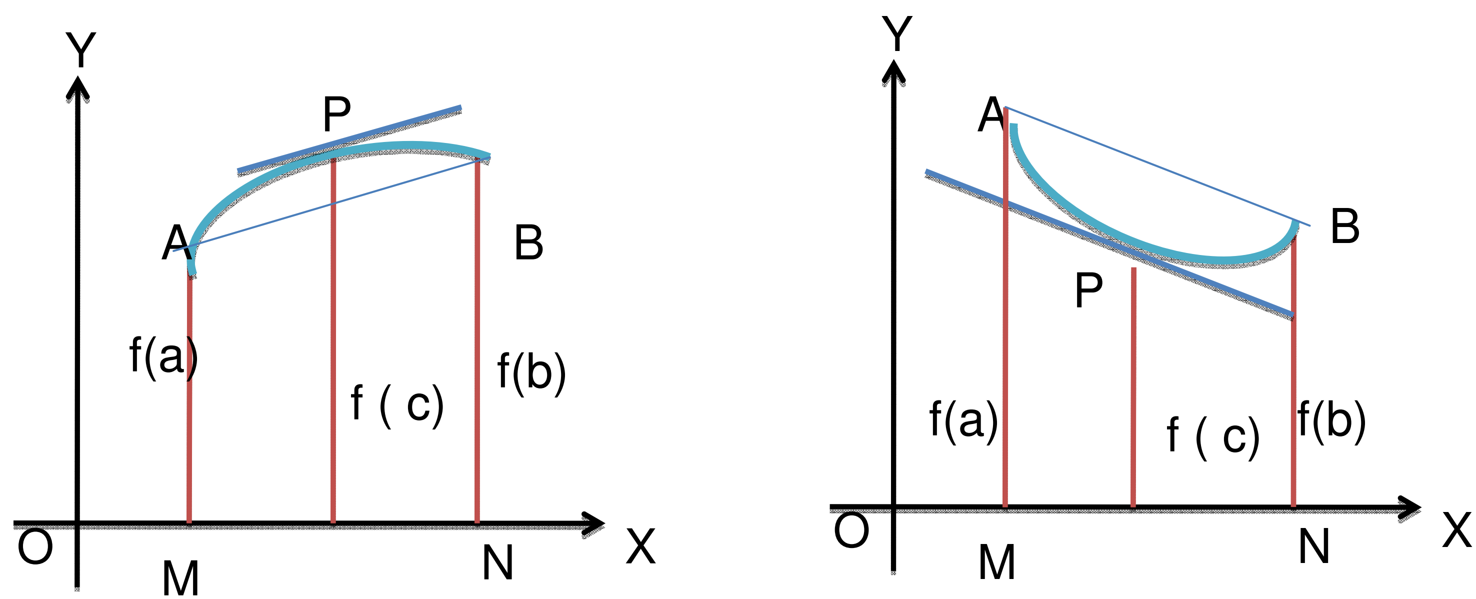
(ii) f(x) is derivable in open interval(a,b)

then there exists at least one real number c (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

L.M.V. Theorem Geometrical Interpretation

Let AB be the graph of function y=f(x) such that the point A and B of the graph correspond to the numbers a and b of the interval [a,b]

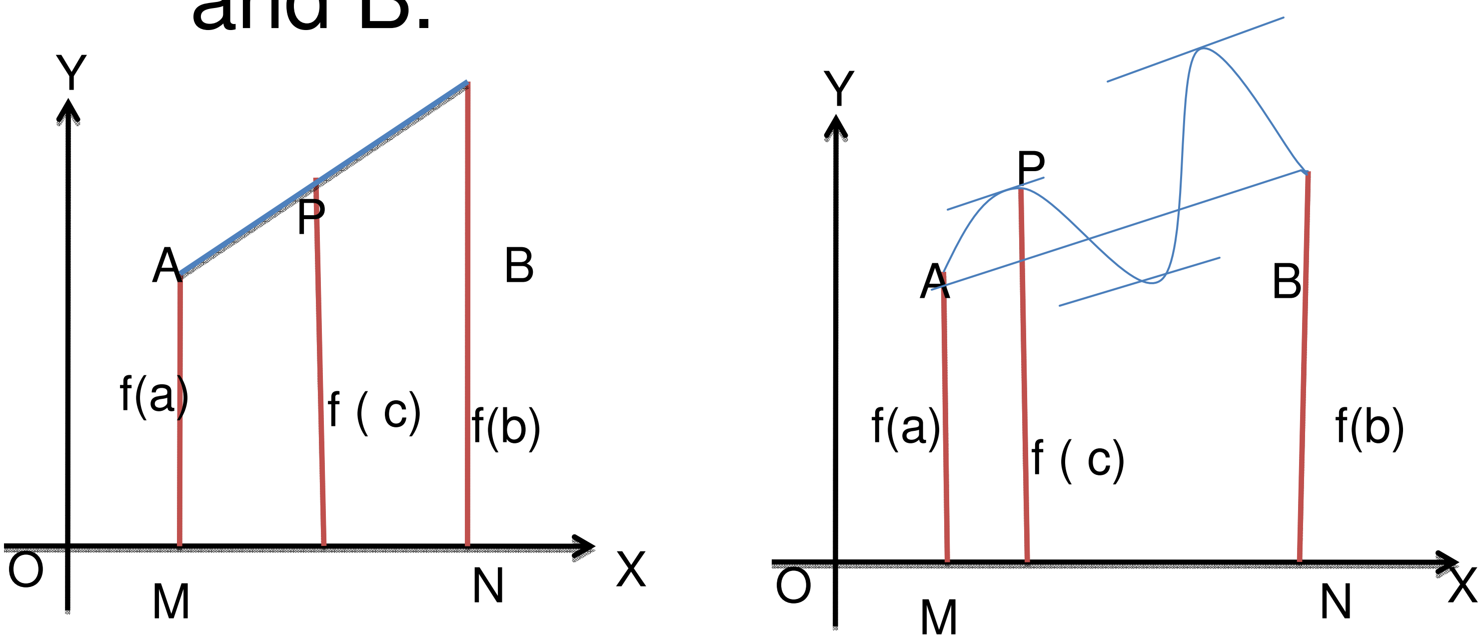


L.M.V. Theorem Geometrical Interpretation

f(x) is continuous in the interval [a,b]

its graph is a continuous curve between A

and B.



L.M.V. Theorem Geometrical Interpretation

Again as f(x) is derivable in open interval (a,b), therefore, graph of f(x) has a unique tangent at every point between A and B.

From four figures it is clear that there is at least one point P on the curve between A and B , the tangent is parallel AB

$\therefore$ Slope of tangent at P= $\frac{f(b) - f(a)}{b - a}$

$\therefore \quad f'(c) = \frac{f(b) - f(a)}{b - a}$,where c is abscissa of P.

Verify L.M.V. in following cases

Q No. 1

$f(x)= (x-3)(x-5)(x-9)$ in interval [3,5]

Q No. 2

$$f(x) = (x-1)^{\frac{2}{3}} \quad \text{on} \quad [1,2]$$

Q No. 3

Find point on the parabola $y=(x-3)^2$ where the tangent is parallel to the chord joining (3,0) and (4,1).

Thanks

- The End