Average Power in AC Circuits

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Average Power in AC circuits

Let the instantaneous voltage be $E = E_0$ Sinωt (or E_0 Cosωt). The resulting current is left out of phase say by an angle , with voltage and as such is given by $I = I_0$ Sin (ωt -ϕ)

The instantaneous power P hence is given by

$$P = V.I = E_0 \, Sin\omega t. \, I_0 \, Sin\,(\omega t - \phi)$$

$$= E_0 \, I_0 \, Sin\omega t \, (Sin\omega t \, Cos\,\phi - Sin\,\phi \, Cos\omega t)$$

Average Power in AC circuits

$= E_0 I_0$ (Sin2ωt. Cos ϕ - Sinωt. Cosωt Sin ϕ)

Or $P = E_0 I_0$ (Sin2ωt. Cos ϕ - (1/2) 2 Sin ωt. Cos ωt Sin ϕ)

$= E_0 I_0$ [Sin2ωt. Cos ϕ - (1/2) Sin2ωt. Sin ϕ]

Average Power in AC circuits

But in A.C. circuits we take Average Power over a complete cycle, Average power therefore is total energy consumed in one cycle, divided by the time period T of the cycle i.e. average power

Average Power in AC circuits

$$P_{av} = \frac{\int_0^T P.dt}{\int_0^T dt} = \frac{\int_0^T P.dt}{[T - 0]} = \frac{1}{T}\int_0^T Pdt$$

$$= \frac{E_0 I_0}{T}\left[\int_0^T \sin^2 \omega t \cos\phi dt - \frac{1}{2}\int_0^T \sin 2\omega t . \sin\phi \, dt\right]$$

Average Power in AC circuits

$$= \frac{E_0 I_0}{T}\left[\cos\phi \int_0^T \sin^2 \omega t \, dt - \frac{1}{2}\sin\phi \int_0^T \sin 2\omega t \, dt\right]$$

$$\text{Now} \int_0^T \text{Sin}^2 \omega t = \int_0^T \frac{1-\text{Cos}2\omega t}{2}.dt = \left[\frac{t}{2} - \frac{\text{Sin}2\omega t}{4\omega}\right]_0^T$$

$$= \left[\left(\frac{T}{2} - \frac{\text{Sin}2\frac{2\pi}{T}.T}{4\omega}\right) - \left(\frac{0}{2} - \frac{\text{Sin}0}{4\omega}\right)\right] = \frac{T}{2}$$

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$$\textit{and} \int_0^T \sin 2\omega t\,dt = \left| -\cos\frac{2\omega t}{2\omega}\right|_0^T$$

$$= \left(\frac{-\cos 2\frac{2\pi}{T}.T}{2\omega}\right) - \left(\frac{-\cos 2.\frac{2\pi}{T}.0}{2\omega}\right) = \left(-\frac{1}{2\omega}\right) - \left(\frac{-1}{2\omega}\right) = 0$$

$$\Rightarrow P_{av} = \frac{E_0 T_0}{T}\left[\mathrm{Cos}\,\phi.\frac{T}{2} - \frac{1}{2}\mathrm{Sin}\,\phi.0\right]$$

Average Power in AC circuits

$$Hence\ P_{av} = \frac{\mathrm{E_0 I_0}}{2}\mathrm{Cos}\,\phi = \frac{\mathrm{E_0 I_0}}{\sqrt{2}\sqrt{2}}\mathrm{Cos}\,\phi$$

$$P_{av} = \mathrm{E_v I_v Cos}\,\phi$$

$\mathrm{Cos}\,\phi$ is called power factor

Power Factor.

- **Here cos ϕ is called power factor**:- When we have a ***pure resistance*** in the circuit, the voltage and current are in the same phase i.e. $\phi = 0^o$ and Cos $\phi = 1$. Equation (3) hence becomes.
- $P_{av} = \mathrm{E}_{\mathrm{eff.}}\mathrm{I}_{\mathrm{eff}}$
- For a pure capacitor or pure inductor in an a.c. circuit, $\phi = \pm 90^o$ and Cos $\phi = 0$.
- i.e. they do not consume any power. Hence are called wattless devices.

Weightage of the topics covered

1.	Electrostatics	-8
2.	Current Electricity	-7
3.	Magnetic Effects and Magnetism	-8
4.	EMI and AC	-8
	Total	-31

44% of the total syllabus has been covered.